THE MATHEMATICS TEACHER

Volume XXXV Number 8

Edited by William David Reeve

Trends in Junior High School Mathematics*

By Raleigh Schorling
University of Michigan, Ann Arbor, Michigan

Part I. Grades Seven and Eight

Some important changes in the teaching of mathematics in these grades during the last quarter of a century are:

1. *Teachers of mathematics are beginning to include materials from the field of aviation in the teaching of the main topics.*

It is fairly clear that we are in a transition period from which we will move into the air age. After the war a vast number of planes and many trained pilots will be available. Great fleets of planes will probably be carrying passengers and express and possibly freight to distant countries. The field of aviation suggests a wealth of applications that are easy to understand and which can be used to introduce and to enliven the topics of the standard curriculum.

2. *General mathematics has replaced arithmetic in grades seven and eight.*

Even the conservative textbooks labeled "arithmetic" introduce informal geometry, make a wider use of graphic materials, and attempt a gradual introduction to symbolism and to a few important concepts of algebra in an effort to teach better the simple and significant principles of arithmetic, algebra, and informal geometry by emphasizing their natural and numerous interrelations.

3. *There is an emphasis on laboratory or workshop techniques.*

This usually takes the form of devoting a part or all of the class to a work- or free-study period in which the teacher (a) observes study habits, (b) checks the results of pupils, and (c) suggests type errors and more effective methods of study. In the workshop setting we use more equipment, investigational techniques, problems involving conscious practice in reflective thinking, techniques for individualization of instruction, and procedures designed to bring about greater socialization.

4. *There is specific emphasis to provide a greater amount of realism.*

In the modern class we try to keep meanings ahead of symbolism. The definitional approach of the arithmetics of an earlier day has been abandoned. In good schools definitions, as well as principles, processes, and concepts, stem from and are summaries of the meaningful number experiences of pupils. The effort to provide realism is further illustrated by the wider use of graphic illustrations, visual and

* Paper read at the Annual Meeting of The National Council of Teachers of Mathematics at San Francisco, February, 1942.

auditory aids, and activities or projects involving measurements and manipulations of materials.

5. *There is a more sensible program of drill.*

With very few exceptions there should be no drill on things that are not clearly understood. However, there are many fundamental skills in mathematics that need to be driven to a high level of proficiency in order that time may not be wasted in learning them in the first place. Some of the suggestions from the psychology of drill that have been extensively applied are (a) drill must *follow* understanding; (b) the pupil should have a desire to learn the things practiced; (c) drill must be individual; (d) it must be specific; (e) drill should be organized so that the pupil may watch his daily growth; and (f) in general it is better to give much practice on a few skills rather than a little practice on many skills.

6. *The problem material is somewhat better.*

We try to provide applications in other subjects and in the various activities of the school, in the home, and in the community. Admitting that problems of the modern texts are far too often absurd enough, problems like the following were included in textbooks of an earlier day, apparently in all seriousness:

> *A human's body, if baked until all moisture is evaporated, is reduced in weight as 1 is to 10. A body that weighs 100 pounds living will weigh how much when dry? (Franklin, 1832.)*

> *What length of cord will fit to tie to a cow's tail, the other end fixed in the ground, to let her have liberty of eating an acre of grass and no more, supposing the cow and tail to be 5 yards and a half. (Walsh, 1840.)*

> *A man and his wife could drink a cask of beer in 20 days, the man drinking half as much again as his wife; but 18/25 of a gallon having leaked away, they found that it only lasted them together for 18 days, and the wife herself for two days longer; how much did it contain when full? (Rev. J. W. Colenso, 1848.)*

7. *In increasing number, teachers of mathematics operate with a broader concept of appraisal.*

One of the achievements of this period is the standardized test. A teacher in the modern school who does not make use of some of the newer testing techniques available, at least for instructional purposes, is either poorly trained or badly misguided. However, there are good reasons for believing that we are entering a new phase of appraisal in which a broader concept of appraisal will challenge the efforts of teachers. Some of the main ideas in this broader concept are that the pupil will have a part in appraisal, and that it will be done in terms of goals that to him seem desirable, definite, immediate, and attainable. A second idea is that we shall try to go beyond scores on standardized tests and will seek to evaluate the intangible outcomes of mathematics by accumulating evidence on such pupil behavior as is socially significant. Finally, we recognize that appraisal is a continuous process by which we need to investigate changes in the same pupils over a longer period of years.

8. *We make greater provision for individual differences.*

The older textbooks did no more than to employ the starred problem; today a book may provide problems on several levels—for example, "easy," "hard," and "still harder." Probably the most effective technique is the use of a project or activity that includes a variety of tasks of such varying difficulty that most pupils can find something that they can do. The work period referred to earlier provides an opportunity for the teacher to confer with superior pupils on special projects, and to examine reports on these differentiated assignments.

9. *In an increasing number of schools there is specific attention in mathematics classes to the reading problem.*

Specialists in remedial reading insist that it is not merely a question of teaching reading as a sort of general skill but that it is necessary for the mathematics teacher to teach the pupil to read a particular type of material. The good teacher (a) provides an experience basis because difficulty in reading is not so much a matter of long words and long sentences as it is unfamiliarity with ideas; (b) begins with the pupil where he is in regard to both his control of the mechanics of reading and his interests; (c) separates remedial reading from the exposition and activity aspects of mathematics; (d) recognizes that if a pupil is to understand a word, a phrase, or a symbol that the course must provide definitely that he lives through some experiences as a basis for the meaning of the word, phrase, or symbol; (e) avoids stigma by not letting a pupil read aloud until he wants to read, but manages the situation in such a way that he will want to read to somebody.

10. *There is more emphasis on specific techniques used in solving verbal problems.*

Among these steps that are given special practice are the following: Judging reasonableness of results in advance of computation; having pupils list what is given and what is to be found; visualizing the conditions by having the pupil draw a sketch when possible; rejecting irrelevant data; recognizing when data are lacking; analyzing relations and selecting processes to be used. The competent teacher, recognizing that a pupil's ability to solve verbal problems is highly tied in with general intelligence and reading ability, will never do today what he can put off until tomorrow, and will confidently expect maturity and experience to come to the rescue.

11. *The teacher of junior high school mathematics accepts a greater burden as regards computation.*

Four trends are clearly characteristic of the arithmetic of grades one to six: (a) There is tendency to omit formal work in grades one and two, and in many schools in grade three. (b) There is a tendency to delay the introduction of a topic to a later grade; for example, a number of schools are no longer teaching per cent in the sixth grade but introduce the topic in the seventh school year. (c) An increasing number of beginning teachers are entering our schools without adequate training in arithmetic and therefore are reluctant to teach, perhaps even fear to teach that which they do not competently control. Then, too, a number of teachers find intriguing the proposal that the arithmetic skills of all six grades be taught incidentally. (d) There is some experimentation with the "core curriculum" in which it seems difficult to include more than a meager smattering of arithmetic. (Since workers with the core curriculum find it exceedingly difficult to include arithmetic experiences in an integrated program dealing with problems of everyday living, teachers are tempted to conclude fallaciously that arithmetic is perhaps after all not so important in living.)

As an indirect outcome of these four trends we find even in conservative schools a reduction in the emphasis given to arithmetic skills; and as a direct result of these trends we find that an enormous additional load has in recent years been added to the secondary school.

12. *Modern courses of study and textbooks emphasize attitudes.*

The report of the National Committee on Mathematical Requirements (1923) was profoundly influential in stimulating makers of courses of study to emphasize such attitudes as annoyance at vagueness and incompleteness, a desire for thoroughness and clearness, an attitude of inquiry, a disposition to discover a general law, and an appreciation of the value of system and organization.

13. *The newer courses emphasize the social implications.*

We recognize today that it is perhaps even more important for the pupil to understand the social implications of such a concept as taxation than it is for him to

do a long list of verbal problems in which he computes taxes. Far more important than the skill to compute taxes is an understanding of taxation as a social or cooperative device for getting more values for one's money in the form of such services as education for children, good roads, delivery of mail, parks and play grounds for recreational purposes, police protection, institutionalizing of the physically diseased and mentally ill.

14. *An increasing number of courses of study list specific objectives for each year or perhaps even each semester.*

A list of detailed items which the pupil is expected to learn at a high level of mastery is of great value alike to pupil, parent, and teacher.

15. *In good schools there is a simplification and a serious effort to limit the processes taught to those that are socially useful.*

In teaching fractions we deal with a few denominators that are common—for example, halves, quarters, eighths. The Report of the National Committee was successful in taking out from the courses of study a number of obsolete topics; for example, complex cases of the highest common factor and lowest common multiple, cube root, and partial payments. We no longer add, subtract, multiply, and divide denominate numbers, and we give more practice on the relations between the various units of measure. Finally, we try to relate ratio, common fractions, decimal fractions, and per cents in a single pyschological unit—for example, $\frac{3}{4}=75/100=.75 = 75$ per cent $=75\%$.

16. *The mechanics of bookmaking have improved.*

The pages in the books are somewhat larger with an effort to provide vivid illustrations and a warm, inviting page by more careful attention to type and illustrations.

17. *The philosophy that experiences in arithmetic should be provided in terms of the individual is gaining acceptance.*

In planning the work we take into account his needs, his stage and pattern of development, his abilities, and interests. We are less concerned by the problem of preparing him for something and more by the need of placing him in an environment where he will develop normally. We have come to see that the factor of readiness for arithmetic experiences is an important consideration.

Part II. Grade Nine

Since the changes relating to general mathematics have been listed in the preceding section, Part II is restricted to the changes in algebra. This list is brief for the reason that the changes stated in Part I apply also to algebra.

1. *Most important of all changes is the general acceptance in the larger schools of the double-track scheme by which general mathematics is offered as an alternative to rigorous courses in algebra.*

Teachers of mathematics are beginning to make the important distinction between the mathematical needs of technical workers and the mathematical needs of general education. It should be noted that in a goodly number of schools the mathematics courses are offered on three levels in this grade.

2. *Algebra has been modified by the general mathematics movement.*

Modern algebra, though it be ever so rigorous, isn't the same algebra as that taught in grandfather's day. For example, there is a more careful approach to symbolism, often by geometric illustrations; the definitional approach has been discarded, graphic techniques are widely applied, new testing techniques are used for instructional purposes, and there has been sharp reduction in manipulation of such technicalities as nests of parentheses, hard cases in factoring and in equations and four-story fractions.

3. *Schools generally have introduced a unit of about four weeks' duration on the triangle (trigonometry).*

In all probability this is the most notable step in the teaching of algebra in the last quarter of a century.* It has not only added realism but has served as a fine illustration of the real purpose of the subject.

4. *The formula has come to be one of the most valuable parts of algebra.*

At the turn of the century the formula was practically ignored in the typical course in algebra. Today the progressive course in algebra begins with the formula, shows its meaning, its practical uses, and the method of deriving one formula from another, and may perhaps introduce it by teaching a unit in which the pupil learns the fourfold method of expressing number relationships. The typical textbook would certainly include a treatment of the formula in the first chapter and might even open with the word "formula" on the first page.

5. *A modern course in first-year algebra strives to develop an appreciation of the function concept.*

The unit on trigonometry was a practical step to achieve this end but it has been looked upon as a kind of unifying principle running through all parts of algebra, to say nothing of its place in the mathematics of grades seven and eight in good schools. The idea is most vividly taught in the treatment of the formula whereby we take such a formula as $v=lwh$, or $i=prt$, and ask the student what happens to one of the factors when certain changes are made in the remaining ones. It is also seen in the treatment of ratio, of fractions, and obviously in proportion. It is doubtful if any modification has done more to give pupils a clearer vision of the real purpose and meaning of algebra.

Since the preceding list aims to include such changes as have been generally made in our schools there is no attempt to provide a complete list of such units as logarithms, slide rule, and statistics which are occasionally taught to enrich first-year algebra. Nor is there here an effort to list all the changes suggested by the two recent committee reports, and by innovations in the work of certain experimental schools. It should not be inferred, however, that such changes, because they are not found in general practice, are unimportant. Indeed the elegant treatment of slope to be found in the first year algebra courses of some schools should perhaps be listed as one of the finest contributions to the meaning of algebra in recent times.

* It should be noted, of course, that a few schools were teaching a unit of trigonometry much earlier than the implied date.

Regional Meeting of the National Council of Teachers of Mathematics

At Stanford University, California, December 28 to 31 Inclusive

THEME: Secondary Mathematics Meets the Emergency and Faces the Future

SUBDIVISIONS:

1. Pre-flight mathematics.
2. Mathematics needed by other branches of the armed services.
3. Industrial mathematics.
4. Consumer mathematics.
5. Teacher training.
6. Pre-engineering and science.

PROGRAM:

December 28	
7:00–8:00	Registration.
8:00–9:30	General Meeting-speech.
December 29	
9:00–10:00	Registration.
10:00–12:00	Panel presentation of six parts of conference.
12:30–2:00	Luncheon-discussion.
2:15–4:15	Group Meetings.
4:30–5:30	Business Meeting.
6:30–9:00	Informal Dinner-speaker.
December 30	
9:00–12:00	Conferences.
12:30–2:00	Luncheon.
2:30–4:30	Panel resume of group meetings.
December 31	
All day	Work Shop.

Recent Trends in Revising the Curricula in Mathematics for Junior High Schools in New York City

By MESMIN ARENWALD
Olinville Jr. High School, New York City

A NEW COURSE of study in arithmetic for Elementary Schools in New York City, Grades 1A–8B, was introduced into the Elementary Schools and into the seventh and eighth year of the Junior High Schools in September, 1929. In the ninth year of the Junior High Schools teachers followed the New York State syllabus in Elementary Algebra. Ninth year students in the Junior High Schools, therefore, followed the course used in the first year of the Senior High Schools and at the end of their ninth year work took the same State examination in Elementary Algebra that was set for Senior High School students at the end of their first year in Senior High School. Junior High School students and Senior High School students no longer take a State examination in Elementary Algebra. Those who complete the work in Intermediate Algebra and pass the State examination in this subject are given credit for both Intermediate and Elementary Algebra.

In 1928, the State Department of Education issued "A Tentative Syllabus in Junior High School Mathematics," in response to a growing demand for a syllabus in Mathematics for Junior High Schools. It was not, however, intended for schools organized on the 8–4 plan.

Junior High Schools in New York City at that time were increasing in number and in response to a growing demand for a distinctive syllabus in mathematics for Junior High Schools in New York City, the Board of Superintendents appointed a committee to draw up such a syllabus. It began its work more than ten years ago. For various administrative reasons its recommendations were not considered by responsible officials until about three years ago.

In February, 1939, twelve typical Elementary Schools and twelve typical Junior High Schools were selected to experiment with the new course of study in the 7A grades. At the end of the term the teachers of the 7A classes in these schools answered a questionnaire based upon their experiences with the new 7A course of study. The answers were carefully studied and revisions in the course of study were made in the light of the criticisms and suggestions that were received.

In successive terms the new course was introduced into the 7B, 8A, 8B, 9A, and 9B classes of the twenty-four selected schools. At the end of each term teachers of the new course answered a questionnaire offering criticisms and suggestions for improving the course. These were evaluated by the committee and the syllabus was modified in the light of the information received.

The questionnaire on the 9A course of study is typical of questionnaires for other grades. The questions are as follows:—

1. a. How have your pupils, the slow as well as the average and the bright, reacted to the new course as compared with the old?
   b. What difficulties did you encounter?
   c. How did you meet them?
2. Give specific recommendations for modifying the 9A course for slow pupils. Give reasons.
   a. Omissions.
   b. Modifications.
   c. Additions.
3. What specific modifications of the tentative course do you suggest for average pupils (95 I.Q.–105 I.Q.). Give reasons.
   a. Omissions.

   b. Modifications.
   c. Additions.
4. What modifications or enrichment do you suggest for rapid classes?
   a. Omissions.
   b. Modifications.
   c. Additions.
5. a. What 9A mathematics classes did you teach during the tryout period?
   b. Give the designation (by approximate median I.Q.) of each class and its register.
6. a. Have you taught the 7A, 7B, 8A, 8B work of the new course of study?
   b. Did your 9A classes have the 7A, 7B, 8A, 8B work of the new course of study?

Pupils who graduate from 8B schools enter the Senior High Schools and the Vocational Schools. A Standing Committee on Mathematics for Senior High Schools and Vocational Schools cooperated with a similar Committee on Mathematics for Junior High Schools before the new 9A and 9B course was introduced in the Junior High Schools as well as the Senior High Schools and the Vocational High Schools. Several members of the High School Committee felt that the proposed course was "too thin" for average pupils and especialy so for bright pupils. On the other hand, the Junior High School Committee pointed to the percentages of failure in Mathematics in Senior High Schools.

The Forty Second Annual Report of the Superintendent of Schools of the City of New York, Statistical Section, School Year 1939–40, shows the following passing rating record in Mathematics for the school year 1939–40: Elementary Algebra—Term 1, 74.1; Term 2, 78.9.

These are the lowest ratings in all subjects including the ratings in Mathematics for other terms. The greatest failure rate is experienced in Mathematics with the rate in Foreign Languages and in Commercial Subjects following in the order mentioned. There are no similar subject rating records available in Mathematics for the Junior High Schools, since very few Junior High Schools promote by subject except in the 9A and 9B. Nevertheless the promotion rate in Junior High Schools for 1939 and 1940 was as follows: Fall Term—94.5, Spring Term—95.1, and has been well over 90% each term since 1926. Standards for promotion of course vary from school to school but it is probably safe to conclude that 90% of Junior High School pupils pass in Mathematics from term to term. The Junior High School Mathematics Curriculum Committee, therefore, feels that the course is not "too thin" if the passing rating record is to be maintained as over 90% in the Junior High Schools. The course may be supplemented, if necessary, and starred topics are introduced that need not be taught to pupils in average classes. These starred topics are most prevalent in the work in Informal Geometry.

In recent years both the Standing Committee on Mathematics for High Schools and the Association of Teachers of Mathematics have formulated or discussed proposed courses for slow learners in the ninth year; a terminal course for non-academically minded pupils in the ninth year; and a basic one-year course for all pupils in all high schools. In general, all pupils would be required to take the basic course but additional work in Academic Mathematics, in Commercial Mathematics or Vocational Mathematics would be elective and restricted to those with the requisite abilities to profit from the advanced work. One or two of these courses have been introduced into a few High Schools. Now that there is no differentiation in courses in Junior High Schools and in the first year of the four year High School, the problem of providing the same basic course in both types of schools becomes a challenging one.

Until a year or two ago the Junior High Schools offered a General Course, a Commercial Course and an Industrial Course. Pupils in the General Course were given arithmetic in the seventh and eighth years and algebra in the ninth year. Pupils in the Commercial Course were given Business Training for a year and Business Arith-

metic for a year. Those in the Industrial Course were taught Industrial Arithmetic. Although there is no course differentiation in Junior High Schools, a half year in Business Training followed by a half year of Business Arithmetic are still offered as electives in many Junior High Schools in place of the usual work in Mathematics. In some Junior High Schools pupils whose work in Mathematics in 7A, 7B, 8A and 8B has been poor and those who are not permitted to take a Foreign Language are given the course in Business Training and in Business Arithmetic. Since there is no Industrial Course, there is no specific course in Industrial Mathematics. Whenever possible, Mathematics is correlated with shopwork. Evidently there is still room for a Junior High School course in Mathematics that may be adapted to the needs of academic-minded pupils as well as to the needs of non-academic-minded pupils. Perhaps adaptation to the needs of bright, average and slow pupils may to some extent solve the problem although many bright pupils prefer concrete problem work in Business Arithmetic to manipulation of symbols in Algebra. Such pupils probably prefer work in Investment, Thrift and Taxes, now in the new 9A course of study for Junior High Schools to work in 9A Algebra.

In introducing Arithmetic into the 9A course of study, the Committee broke the New York City tradition of teaching Arithmetic in the seventh and eighth years and Algebra in the ninth year. In this respect the Committee felt that ninth year pupils would profit from the study of Arithmetic correlated not only with Algebra but with Civics and with other subjects taught in the ninth year.

How did the Junior High School Committee go about its work?

On the whole it retained the 1929 Arithmetic syllabus followed in the seventh and eighth years. It studied textbooks in the field, both city and state syllabuses, Year Books of the National Council of Teachers of Mathematics, the 1923 Report of the National Committee on Mathematical Requirements, the 1940 Report of the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics and finally the 1940 Report of the Progressive Education Association. Of course, publications in the field of curricular research were read very carefully. It should be borne in mind that the members of the Committee were not relieved from supervisory or teaching assignments to enable them to concentrate on the revision of the Junior High School Curriculum in Mathematics. They were given very little clerical assistance. In addition, the Committee was concerned with the introduction of the new course and therefore with the attitude of the teachers towards it. To orient them properly one member of the Curriculum Committee conducted an In-Service Course for teachers and over five hundred teachers have already availed themselves of the opportunity to take this course. The need for such a course is apparent because many who teach Mathematics in the Junior High Schools are not licensed in that subject and many who have been teaching the subject for a long time have not kept abreast of modern tendencies in the teaching of mathematics.

How does the new course differ from the old course?

As previously stated, the work in Arithmetic in the new course is practically the same as the work in Arithmetic in the old course although the grade placement of topics is different in the two courses. In the new course Arithmetic is taught in the 9A grade.

Secondly, work in Informal Geometry is begun in the 7A and is continued through all grades of the Junior High School. In the old course, Informal Geometry was taken up only 8B. In the new course, the work in Algebra is begun in 8A and is continued in 8B, 9A and 9B. In the old course, only simple algebraic equations applied in the solution of the problems of the grade were taught in the

8A and 8B. Very few teachers, however, taught this topic except to bright groups. By including Arithmetic, Informal Geometry and Algebra in the new course of study, and by making provision for supplementing and modifying the course from time to time, the committee felt that its recommendations were in line with prevailing tendencies in curriculum reconstruction.

Although the introductory notes and footnotes of the new course suggest methods of teaching individual topics, the new course is essentially an outline of large topics to be taught and the syllabus contains sub-topics under each topic in the course of study. It is, therefore, not a handbook similar to the booklet "Mathematics for Elementary Schools" issued by the State Department of Education. The Curriculum Committee could not find time to develop a handbook that resembles a textbook in methods of teaching. In fact, the committee felt that the primary function of supervisors was to improve the quality of instruction through conferences, through observations and through demonstration lessons. A handbook may be helpful if personal supervision is limited. This, however, is theoretically not the case in New York City.

The idea of an "Integrated Curriculum" is not new to education. The Virginia course of study with its "Centers of Interests" is an excellent illustration of an "Integrated Curriculum." In New York City such a curriculum has been tried in the Elementary Schools in what is known as the "Activity Program" and as an "Integrated Curriculum" in one of the larger academic High Schools.

On the one hand, the new course of study for Junior High Schools in Mathematics is not part of an integrated Junior High School curriculum. On the other hand, the report of the Progressive Education Association on "Mathematics in General Education"—"does not recommend a single more or less formal course of study but outlines a set of fundamental concepts and guiding principles designed to serve as a basis upon which teachers may so organize their own work as to make it appropriate to the possibilities and limitations of individual schools or classes, or ideally, individual students." Are teachers professionally prepared to carry out this recommendation of the Progressive Education Association? If not, then a course of study similar to the new Junior High School course in Mathematics in New York City is most practicable under the circumstances. In it specific topics in Arithmetic, Informal Geometry and Algebra, as well as desirable outcomes, are indicated for each grade. In many respects this new course of study is similar to the one suggested by the Joint Commission on pages 82–91 of its report on "The Place of Mathematics in Secondary Education." The grade placement of topics is not identical in the two courses but the required and optional topics are practically identical. The considerations governing the selection of the materials of instruction and the principles of arrangement followed by the Joint Commission are also basically the same as those used by the New York City Curriculum Committee.

The course the Committee has drawn up is meant primarily for average pupils of 95 I.Q. to 105 I.Q. A mimeographed supplement is provided for teachers of slow classes and another mimeographed supplement is provided for teachers of bright classes. These guides include principles, general notes, deletions, additions and modifications in the course for average pupils to adapt the course to the capacities of slow pupils or of bright pupils as the case may be. In addition, a special report on enrichment of the 7A course in Mathematics for bright pupils, based on the suggestions of teachers of such classes, has been sent to all teachers of bright 7A classes in Mathematics. Here again the suggestions of the Curriculum Committee are similar to those expressed in Chapter VII—"The Problems of Retardation and Acceleration" in the Fifteenth Yearbook

of the National Council of Teachers of Mathematics. Many topics in the report of the Joint Commission are discussed in the following introductory topics of the Junior High School course of study and syllabus:—Underlying Philosophy, General Aims, Specific Objectives, Individual Differences, Skills, Notes on Drill, Problem Solution, the Testing Program, Notes on Arithmetic for All Grades, Notes on Informal Geometry for All Grades, Notes on Algebra for All Grades and Checking.

Now that the new course of study has been approved by the Board of Superintendents and the Board of Education, the Curriculum Committee feels that New York City Junior High Schools have a well-organized modern course in Mathematics that is flexible, adaptable and continous from the 7th through the 9th years. Supervisors are helping teachers to interpret and to teach the new course. Last term, demonstration lessons were presented to groups of teachers interested in teaching the new course properly. Many of the older textbooks will have to be replaced by new books that reflect the philosophy and objectives of the new course of study. New classroom equipment to teach mathematics properly will have to be made available. Teachers and supervisors are cooperating wholeheartedly to solve the common problems incident to introducing the new course. They are rapidly acquiring a new philosophy and a new incentive to improving the quality of the instruction in Mathematics. They realize that only through such re-orientation will they be able to benefit all Junior High School pupils who study mathematics and succeed in encouraging many to elect more advanced work in mathematics when they enter Senior High Schools or Vocational High Schools. These objectives can be attained when pupils realize that mathematical thinking developes a mode of thought that is needed to help solve the problems of living in our democratic society.

Because of the war, interest in mathematics and its applications has increased markedly. It is now reflected in the change of emphasis in the teaching of Junior High School Mathematics. Teachers in New York City are using a report prepared by the Junior High School Mathematics Workshop which contains a wealth of illustrative problems relating to the war effort. They have not found it necessary to omit any topic in the course of study. At the same time they agree that the general aims and specific objectives as formulated before we entered the war, need not be changed. The impact of the war has merely enhanced the appreciation of their worth and importance.

FOR SALE

Complete set of

1. *School Science and Mathematics*
Bound in Red Cloth

2. *American Mathematical Monthly*
Bound in Blue Cloth

Also one set of unbound volumes at $2 each of *The Mathematics Teacher* for the year 1926 to 1941 inclusive

For further particulars write to

The Mathematics Teacher, 525 West 120th St., New York, N.Y.

Vitalizing Junior High School Mathematics for Non-College Students*

By Mary Rogers
Westfield, N. J.

It is interesting to note that in contrast to the now apparent decrease in Elementary School enrollments, our Secondary Schools continue to grow. Statistics show an *increase* of nearly 2,000,000 pupils during the *past decade*, to an all time high of 6,650,000 in 1940–41. We are told that the tendency for an increasingly large *proportion* of our young people to remain in school through the Senior High School will doubtless maintain this high enrollment for some time to come.

Many of these students show very little interest in *academic* training, both as a result of natural heritage and of environmental influence. They find the more formal education too difficult, and anticipate very little use for it in later life. For them the social implications of their immediate environment are far more meaningful and so seem to them of more vital importance. How can we provide in our curricula experiences vital to this type of child?

It is this challenge which we have kept in mind in working out the activity with non-college ninth grade pupils, which I am presenting to you today.

I do not presume to suggest that our way of providing for these needs in Westfield should form a criterion for other school systems, or be followed literally in any sense. There are certain sociological conditions universally prevalent; furthermore, certain general psychological principles, studied and carefully applied, will avail much to anyone teaching this type of child. But, just as surely as individual differences exist among persons, just so surely, many differences are apparent among communities—differences in economic, political, social and religious life—differences caused, also, by tradition and custom. So each community must work out its own program. But, if in meetings such as this, we may, by an exchange of experiences, offer any suggestions of mutual benefit to us all, we shall have made our small contribution toward educational progress.

Westfield is a residential community of about 18,000 population. We have *no* industries, and only such business organizations as cater directly to the wants and needs of our own people. A very large per cent of our men and women are college and university trained. This class of person, engaged in gainful occupation, *largely* finds employment in New York City filling executive or professional positions. Our foreign and colored population finds local employment in mercantile business, on street construction and maintenance, doing gardening, domestic labor and the like.

About 65% of our ninth grade pupils take College Preparatory work. It is with the other 35% that we do the type of work which I am about to describe to you. Many of these children are intellectually retarded or socially mal-adjusted, but most of them do have average comforts of life at home, and do enjoy some of the same pleasures, for instance, driving the family car at the earliest possible age.

Automobile driving is one of the anticipated delights of *every* Twentieth Century youngster. He looks forward with the keenest delight to the time when he can sit behind the wheel of a car and feel the thrill of the power at his control. Youth craves adventure, swift action, excitement, changing scenes, untried experi-

* Speech presented at the Summer meeting of the National Council of Teachers of Mathematics, July 1, 1941.

of the National Council of Teachers of Mathematics. Many topics in the report of the Joint Commission are discussed in the following introductory topics of the Junior High School course of study and syllabus:—Underlying Philosophy, General Aims, Specific Objectives, Individual Differences, Skills, Notes on Drill, Problem Solution, the Testing Program, Notes on Arithmetic for All Grades, Notes on Informal Geometry for All Grades, Notes on Algebra for All Grades and Checking.

Now that the new course of study has been approved by the Board of Superintendents and the Board of Education, the Curriculum Committee feels that New York City Junior High Schools have a well-organized modern course in Mathematics that is flexible, adaptable and continous from the 7th through the 9th years. Supervisors are helping teachers to interpret and to teach the new course. Last term, demonstration lessons were presented to groups of teachers interested in teaching the new course properly. Many of the older textbooks will have to be replaced by new books that reflect the philosophy and objectives of the new course of study. New classroom equipment to teach mathematics properly will have to be made available. Teachers and supervisors are cooperating wholeheartedly to solve the common problems incident to introducing the new course. They are rapidly acquiring a new philosophy and a new incentive to improving the quality of the instruction in Mathematics. They realize that only through such re-orientation will they be able to benefit all Junior High School pupils who study mathematics and succeed in encouraging many to elect more advanced work in mathematics when they enter Senior High Schools or Vocational High Schools. These objectives can be attained when pupils realize that mathematical thinking developes a mode of thought that is needed to help solve the problems of living in our democratic society.

Because of the war, interest in mathematics and its applications has increased markedly. It is now reflected in the change of emphasis in the teaching of Junior High School Mathematics. Teachers in New York City are using a report prepared by the Junior High School Mathematics Workshop which contains a wealth of illustrative problems relating to the war effort. They have not found it necessary to omit any topic in the course of study. At the same time they agree that the general aims and specific objectives as formulated before we entered the war, need not be changed. The impact of the war has merely enhanced the appreciation of their worth and importance.

FOR SALE

Complete set of

1. *School Science and Mathematics*
   Bound in Red Cloth

2. *American Mathematical Monthly*
   Bound in Blue Cloth

Also one set of unbound volumes at $2 each of *The Mathematics Teacher* for the year 1926 to 1941 inclusive

For further particulars write to

The Mathematics Teacher, 525 West 120th St., New York, N.Y.

Please mention the MATHEMATICS TEACHER *when answering advertisements*

Vitalizing Junior High School Mathematics for Non-College Students*

By Mary Rogers
Westfield, N. J.

It is interesting to note that in contrast to the now apparent decrease in Elementary School enrollments, our Secondary Schools continue to grow. Statistics show an *increase* of nearly 2,000,000 pupils during the *past decade*, to an all time high of 6,650,000 in 1940–41. We are told that the tendency for an increasingly large *proportion* of our young people to remain in school through the Senior High School will doubtless maintain this high enrollment for some time to come.

Many of these students show very little interest in *academic* training, both as a result of natural heritage and of environmental influence. They find the more formal education too difficult, and anticipate very little use for it in later life. For them the social implications of their immediate environment are far more meaningful and so seem to them of more vital importance. How can we provide in our curricula experiences vital to this type of child?

It is this challenge which we have kept in mind in working out the activity with non-college ninth grade pupils, which I am presenting to you today.

I do not presume to suggest that our way of providing for these needs in Westfield should form a criterion for other school systems, or be followed literally in any sense. There are certain sociological conditions universally prevalent; furthermore, certain general psychological principles, studied and carefully applied, will avail much to anyone teaching this type of child. But, just as surely as individual differences exist among persons, just so surely, many differences are apparent among communities—differences in economic, political, social and religious life—differences caused, also, by tradition and custom. So each community must work out its own program. But, if in meetings such as this, we may, by an exchange of experiences, offer any suggestions of mutual benefit to us all, we shall have made our small contribution toward educational progress.

Westfield is a residential community of about 18,000 population. We have *no* industries, and only such business organizations as cater directly to the wants and needs of our own people. A very large per cent of our men and women are college and university trained. This class of person, engaged in gainful occupation, *largely* finds employment in New York City filling executive or professional positions. Our foreign and colored population finds local employment in mercantile business, on street construction and maintenance, doing gardening, domestic labor and the like.

About 65% of our ninth grade pupils take College Preparatory work. It is with the other 35% that we do the type of work which I am about to describe to you. Many of these children are intellectually retarded or socially mal-adjusted, but most of them do have average comforts of life at home, and do enjoy some of the same pleasures, for instance, driving the family car at the earliest possible age.

Automobile driving is one of the anticipated delights of *every* Twentieth Century youngster. He looks forward with the keenest delight to the time when he can sit behind the wheel of a car and feel the thrill of the power at his control. Youth craves adventure, swift action, excitement, changing scenes, untried experi-

* Speech presented at the Summer meeting of the National Council of Teachers of Mathematics, July 1, 1941.

ences. To him the automobile brings all these abundantly. Small wonder he responds so enthusiastically to its appeal. When questioned, he concedes there is much he does not know about this absorbing interest; but he is very eager to learn. Would we be *willing* to help him?

We accept this challenge. Here is a course along which education and life run *closely* together—an opportunity to assist in the acquisition of practical and useful information, to clarify understanding of social environmental factors, to promote proper attitudes toward civic rights and responsibilities. And what an opportunity to teach facts, skills, and concepts which have a direct transfer value!

We organize this *Know Your Automobile* unit as a *correlated* study, running simultaneously in the Mathematics, Social Science, General Science and English classes of *all ninth* grade non-college students. We stress Safety, but we *do* admit the adventure of the automobile.

We plan the work for late Winter or early Spring when the lure of the road is again strongly apparent and the week end pleasure jaunt is the common topic of discussion in the home. Informally, out of class, the children are *encouraged* to talk automobiles. The teachers engage in these conversations whenever possible, thus ascertaining the types of interests displayed by these youngsters. Then, on a certain day, which marks the beginning of this project, the children coming to English class find the bulletin boards covered with pictures of unnamed automobiles. A contest is launched in recognizing and naming these different cars. As you will surmise, this meets with great enthusiasm—here is an opportunity for self-expression on a subject really meaningful to them. Of course, nearly all the boys know all the cars—many of the girls do nearly as well. An animated discussion just naturally breaks forth, as to the relative merits of the various cars on display. The teacher enters into the interest of the occasion, recognizing the opinions expressed, but urging that *facts* be assembled with which to substantiate these statements. The children are advised that much worthwhile material may be obtained from local dealers if a business-like procedure is followed. The class is organized into groups and plans for study outlined. The correct method of interview is discussed and criticized, and practice afforded therein.

There follow several days devoted to examining circulars and pamphlets contributed by the dealers, and in reading advertisements in magazines and newspapers. The children learn to select pertinent facts, and to discount those whose main purpose is to attract attention. There is developed a wholesomely critical appreciation of advertising. Furthermore, much practice is afforded in written and spoken expression. This study is known as The Appeal of the Advertisement.

During this study, the boys have become interested in data relative to the mechanism of the car. The English teacher frankly confesses her inability to *explain* these things and refers the class to their Science teacher.

In the Science class plans have already been set up for the continuation of this project. Through well chosen reference material ("Man and the Motor Car" and Buick Manual, General Science Texts), films, (The *Story of the Gasoline Motor* and The *Power Within*) and charts, the children become acquainted with the structure of the gasoline engine and the principle involved in the generation of power. They learn the significance of the clutch. They observe the workings of the transmission system. They trace the flow of power from the cylinder to the rear wheel. They study proper lubrication and come to appreciate its importance to the smooth running efficiency of the automobile. They are shown the marvels of the electric system. They become acquainted with many safety devices; they learn the science of their structure and observe precautions to be taken in their use. It should

be a revelation to these youngsters to know just how *safe* this powerful modern vehicle *can* be, if only it is properly controlled.—They are to hear more about this later in their Mathematics classes.

In every class, there are *boys*—even at this level—who are keenly interested in every detail of mechanics that is available. To them is given an opportunity to make a *special* study of the valve-in-head engine, the electromatic clutch, sealed beam head lights and many other devices featured in the new cars. Much material is available from the General Motors Company, Automobile Manufacturers Association, Building America and similar sources, for these studies and for many other simple problems of research. The boys enjoy this material and talk about it as enthusiastically as they do articles in current issues of *Popular Science Monthly* and *Popular Mechanics*.

In the meantime, a chart showing the development of self-propelled vehicles has been displayed in the Social Science classes, and has caused considerable surprise. It seems incredible that as early as 1600 A.D., a vehicle successfully travelled, propelled by its own power. This sailing chariot, known earlier in China, and brought to its peak in Holland in 1600 by Simon Stevin actually covered forty-two miles in two hours, carrying twenty-eight persons; for a time it was used quite regularly. Nor was this the beginning of man's quest for more speed, more thrills, more wealth. Even as early as 130 B.C.—the class learns—one Hero of Alexandria invented and used a steam propelled vehicle known as the Aeolipile. It was not until the beginning of the eighteenth century, however, that man began to think seriously of power transport, began to experiment and to build constructively toward the motor age we now enjoy.

Much progress was made during the nineteenth century; the differential, three speed transmission and gasoline driven motors made their appearance during this time. But the automobile age did not really arrive until the beginning of the twentieth century. In the short space of one generation this vehicle has changed not only man's manner of living but his entire outlook on life. The children have become aware of these social implications through simple narrative, well chosen motion pictures, such as the *Harvest of the Years* and conversation with persons who have lived through these exciting changes.

They are not at all suprised, to discover in the Mathematics classes, how very largely the motor industry contributes toward the maintenance of other major industries. It is interesting but not amazng to learn that

1. 90% of all gasoline used in the United States drives the motor vehicles of the country.

2. That 80% of all our rubber finds its way into automobile construction.

3. That 75% of all plate glass, 55% of alloy steel, 51% of malleable iron, 68% of leather contribute to the manufacture of these vehicles.

The children estimate quite accurately the ratio of passenger cars to population and are eager to check their estimates with established facts. An advance bulletin from the Automobile Manufacturers Association tells us that 27,300,000 passenger cars were registered in 1940; the latest *Americana Annual* gives the 1940 population as 131,669,275. These children love to round off numbers, and do so very accurately with a little guidance. We decide to read these numbers by millions and set up a ratio of 27 to 132. This simplifies to 9 to 44 or very closely approaches 1 to 5. A passenger car to every 5 *persons* in the country! Perhaps curtailing production to speed up the defense program will not work too great a hardship.

It is a little difficult to believe that an average of 1 out of every 20 persons in the United States is directly employed in the automobile industry—in manufacturing, sales and servicing, truck driving, petroleum refining and highway maintenance

—but here are the figures for 1940—6½ millions so employed from a population of about 132 millions. *Automobile Facts and Figures* for 1940 help us discover that motorists, through registration fee, licenses, gasoline and motor taxes contribute \$1.00 out of every \$9.00 for all government support—in normal times. Our figures state \$1,636,549,000 *motor* tax with an estimate of \$13,800,000,000 total revenue.

Here are other facts furnished by the Metropolitan Life Insurance Company which are quite as large in quantitative values but much less cheerful in social implications. We are confronted with the statements "Automobile accidents take 100 lives a day . . . 150 are *injured* every hour . . . auto accidents cost \$3000 each minute." Can these statements be true? Let us see what this little book has to say about it. The Travelers Insurance pamphlets are always popular with the children. They are so profusely illustrated; the material is so diversified, and so well suited to this age and class of students.

Yes, here we have it—page 3—1940's death toll 35,000—injuries 1,320,000. There were 366 days in 1940; if we divide 35,000 by 366 we find that fatalities did average about 96 each day—not quite 100 but very nearly so. For each of the 8784 hours in 1940 we find there were more than 150 injuries.

Another pamphlet gives information as to costs. We read "Estimates of the total economic loss from motor accidents which include the value of damaged property, wages lost by injured workmen, medical and hospital fees, range from one and a half billion dollars annually to two and a half billion dollars." There were 8784×60 minutes or 527,040 minutes; at the \$3000 a minute estimate we find an annual accident cost of \$1,581,120,000! What a waste! If this could only be saved, it would go a long way toward new safe super-highways or other social improvements!

We are told *Man causes* all this disaster—always The Other Fellow. We are also told he is generally an experienced *driver*. But we do not accept his indictment until we have built up a case against him. Perhaps you—or I—may also be at fault. Booklets from the Travelers Insurance Company—also Accident Facts, the annual report from the National Safety Council—are filled with statistics, incontestable facts—if only we can make them live, paint us pictures. Some one suggests graphs. (These children have made simple graphs in the seventh and eighth grades and have enjoyed them very much. Furthermore, earlier this year, we have been reading graphs about distribution of national income, relative living standards, etc.—this in connection with out study of percentage.) It is not difficult then to interest them in graphing these appalling accident facts.

First, we study the most common *types* of accidents, and seek to determine what part of the total of all fatalities is attributable to each of these types. This calls for a circle graph. We copy these facts in tabular form, figure the percents of all accidents represented by each type shown and determine the number of degrees to be laid off in each sector of the graph. Then we proceed to construct, name and label the graph. Attention is called to the fact that these protractor measurements must be very accurately made, for some of these accidents are relatively infrequent and will not show at all in our graph unless we are very careful.

We find we are as yet ready to pass very little judgment. It *is* interesting to note, however, how strongly another person—the pedestrian—is implicated in these happenings.

This pedestrian may mean *you*—or *me*, we decide, so we study *his* actions rather closely. Apparently he cannot or *will* not take care of himself, for nearly all fatalities occur when he is left to himself, unguarded by traffic safeguards. Frequently, serious accidents occur when he attempts to cross *between* intersections; when at an *unguarded* intersection, he vies with auto-

mobiles for a disputed right of way. The *rural highway continues* to take a heavy toll of these careless individuals also. We wonder whether *still* there are people who do not know that pedestrians travelling along motor highways *always* should *face* oncoming traffic. Many of these deaths occur at night. Pedestrians should wear light colored clothing or carry some lighting device if the must walk abroad at night.

We observe that the pedestrian has some difficulty at *guarded* intersections because he *will* proceed against the signal. Perhaps he does not understand that the yellow light means *clear* the intersection. Neither driver nor pedestrian should advance on this light unless caught within the intersection when the green light changes. Some cities display pedestrian signs as good reminders of this warning.

This has been about our first story telling from graphs and we are eager to check the accuracy of our interpretation. Motion pictures are always fun and very easy to understand. It is remarkable how accurately the film Man on Horseback verified out observations. Pedestrian Habits was helpful, too, in that it concerned Junior High School people and pointed out some of *our* mistakes.

We are now ready to return to the driver. To him, we recall, has been ascribed the fault for all this traffic disaster. We have already exonerated him of some of the blame, but there is plenty yet to be considered. We make our computations and construct the graph very carefully so that again our story may tell the truth. We must concede that the evidence seems pretty strongly against him; nearly one-half of all deaths have been caused by his excessive speed; much reckless driving is apparent here also—and disregard of accepted codes of right of way. But is *controlled* speed necessarily a misdemeanor? Perhaps the condition of the automobile makes control difficult. That is not apt to be so in New Jersey, however, for all cars *must* pass inspection *twice each* year. Here are facts; as we picture them. Apparently the car *nearly always* is in perfect condition when accidents occur. Brakes and lights are *occasionally* faulty and even less frequently some other piece of mechanism fails to function, but such occurrences *are not* frequent.

But we *still* have not considered highway and weather conditions and general conduct in traffic and they should afford us very interesting information. We are concerned here with *relative numbers* of accidents, so we decided to use bar graphs. Bar graphs are fun to make if you round off the numbers and use decimally coordinated graph paper for the construction work. Again we make tables of values. We note our largest value to be shown; we study our graph paper for size; we plan attractive margins; we determine a suitable scale, always so many *hundreds* or *thousands* to the inch so that our bars may be measured by the units on the graph paper. A ruler is not needed in constructing the bars; a straight edge does just as well. Of course, we complete our table, filling in lengths to be measured, before we begin graphical construction. The completed graphs are named and labeled to agree with the tabulated data, and we are ready to show you our pictures,—to tell the stories.

These prove to be *interesting* and rather *unexpected!* Nearly all deaths occur when the weather is *clear* and the highways *dry*. The driver is usually going *straight ahead;* most frequently he is on an open *straight stretch—between* city intersections or on rural highways. Apparently the driver believes a long, fine stretch of pavement is perfectly *safe;* he "steps on it" and fails to observe the rules of safe driving.

Again we watch the verification of our conclusions by motion picture. Paramount Pictures Inc. puts out a picture called And Sudden Death. This we think is one of the most complete reproductions available in the field of highway safety—a most convincing story of enforcement difficulties.

Despite existing conditions law enforce-

ment officials *do* concede that many drivers' faults are unintentional. In many States, licenses are relatively easy to obtain. The driver is not *trained* to drive; he does not know the Codes of the Road, does not appreciate the natural forces at work, does not know his own limitations, nor the power of the machine to be controlled.

Engineers may produce better automobiles and build safer highways; highway officers may attempt to enforce laws, but until our drivers become better *educated*, accidents will persist.

I shall tell you of a few of the studies we are making. We do not attempt to teach the techniques of Safe Driving. That training comes to these young people later in the Senior High School. *We* try to build a *background* of appreciations.

First, we study the Codes of the Road. The Department of Motor Vehicles at Trenton generously furnishes us with as many copies of this Manual as we can use in our classes. The children are eager to get them—to discover how much they *already* know and to learn more. The high point of this particular study is the quiz program, patterned after the Quiz Kids or some other popular radio performance. Each student participates in a preliminary elimination contest by means of which final contestants are chosen. Interest is keen and much benefit is derived from this activity in the Social Science classes.

In the meantime, much interesting work is going on in the Mathematics classes. We have conceded that speed is not necessarily a misdemeanor, but uncontrolled speed surely is, at least, a *threat* to safety. Seeming recklessness may be ignorance or misunderstanding.

The children are keenly interested in the experiments and problems with which we clarify this understanding. The study involves, again and again, rates of speed—miles per hour of travel. We soon find that although we are accustomed to reading the speedometer of an automobile, miles per hour means very little to us, so we make a chart showing the corresponding number of feet per second—average number of car lengths per second.

We often hear the remark, "It all happened before I could think." Isn't this quite possible? There is such a thing as reaction time—the time it takes to think and to convert that thought into action. It varies among individuals but the average time is generally given as $\frac{3}{4}$ sec. At 60 miles an hour, we see, a car could go *66* ft. or about *4 car lengths* in an emergency before the average person could *act*.

Always eager for games or contests, the children enjoy thoroughly the simple experiments in the little book—*Fun with Facts*—put out by the Travelers Insurance Company. A simple game but full of meaning for us is the coin dropping experiment. One person drops a coin from various heights while a second person attempts to intercept it before it hits the floor. Each person has several attempts at each of the different heights, keeping score of his hits at each height. He notes the height at which his score is most successful and by applying the simple formula $S=\frac{1}{2}gt^2$, he determines his approximate reaction time. The boys are always certain they will be better than the girls at this game, and they generally are.

This year we were able to secure a *reactometer* from the Actna Casualty and Surety Company. The children have enjoyed using a commercial testing instrument; furthermore their reaction score is so much more accurate. After each person is tested, he is given his score card and a little time card on which reaction time is converted to feet of automobile travel per second, at varying rates of speed. The children were quite pleased with their scores.

Reaction distance + braking distance = stopping distance. But what can we expect of brakes at varying rates of speed and under different road conditions? A chart, furnished by General Motors Company pictures this very vividly, and proves very useful. We decide, however, that an idea, clearly understood, is *more* useful than a chart. We compare increased brak-

ing distances with increased speeds. We set up ratios and compare these ratios.

M.P.H.	*Braking Distances*
20	22.2 ft. per sec.
40	88.8 ft. per sec.
60	199.8 ft. per sec.

Speed Ratios	*Braking Ratios*
$\frac{40}{20}=\frac{2}{1}$	$\frac{88.8}{22.2}=\frac{4}{1}$
$\frac{60}{20}=\frac{3}{1}$	$\frac{199.8}{22.2}=\frac{9}{1}$

With very careful guidance, we discover that the braking distance does change *directly* with changing speeds, but not with equal ratios. The braking distances lengthen directly as the *square* of the increasing speeds and that is quite a different matter. Attention is called also to the effect of wet or otherwise slippery highway on stopping distances. When *friction* is reduced—decreased—stopping distances are considerably lengthened. There follows a short period of comparison of stopping distances at various rates of speed with known distances about the school grounds. Some incredulity is always voiced, but these children have been stimulated to observe—to *think*.

This knowledge of stopping distances is *so* essential to proper spacing in fast, heavy traffic and yet how few ever really think about it! A driver surely takes chances when he drives *so close* to the car ahead that he cannot,—in an emergency—stop within that distance. What, under average conditions, are safe spacing distances at different rates of speed? The children study the chart and rather reluctantly admit the following: If you travel at

M.P.H.		*Ft. spacing*		*Car Lengths*
30	allow	80	or	5
40	allow	130	or	8
50	allow	195	or	12
60	allow	270	or	16

this last distance being about one-half again as much as the length of our Junior High School building in Westfield.

"But," some one exclaims "who *ever* observes these distances? Wouldn't some one squeeze in ahead of you if you tried?" "Yes, quite likely" we agree, "especially in heavy week end or holiday traffic when the driver is in a hurry and finds passing difficult."

"What is a safe passing distance?" we are asked.

"That is rather difficult to determine," we answer. "So many factors enter into the situation, but perhaps we can work out something that will at least challenge your closer attention in future experiences." Here is an interesting question in this list of automobile problems sent out by the Travelers Insurance Company.

"How long will it take car A which is 18 ft. long and going 45 M.P.H. to pass car B which is 16 ft. long and going 40 M.P.H.? (Figure from time car A's front is abreast car B's rear, to the time when car A's rear is abreast car B's front.)"

First we draw a diagram and note that A must travel at least 34 ft. more than B. Let's represent B's distance as D. Then A travels $D+34$ ft. The time each travels *must* be the same. B travels 40 M.P.H. or by our chart about 59 ft. per second; A travels 45 M.P.H. or 66 ft. per second. These children are quite familiar with their distance, rate, time formula—so it is not *too* difficult to set up this equation

$$\frac{D}{59}=\frac{D+34}{66}.$$

Solving we find D or B's distance to be 286 ft. or about 18 car lengths. A would travel 20 car lengths. He should allow *double* that distance between himself and the car approaching in the other traffic lane for that car is doubtless travelling at least at his own rate of speed. Forty car lengths are about 1/10 mile. You have often "clocked" 1/10 mile on your dad's speedometer, we tell the children but "clock" it again if you have forgotten. We work out several more such problems, changing the rate of speed of either or both cars, and noting how this affects the space to be allowed. We always emphasize that

the car approaching is a most uncertain factor; so allow for him generously.

The children have heard much about the heavy toll of automobile fatalities over the week ends and are interested in verifying these reports. Accordingly they again graph facts discovered in the Travelers Insurance pamphlet. Apparently there have been few exaggerations. Comparison of this graph with a similar one made several years ago shows a persistence of these conditions. Perhaps we are depending too greatly upon our automobiles for amusement and play. On the other hand, not all Sunday driving is for recreation. A very interesting article brought to class by one of the children, says "Upwards of 6,000,000 motor cars are employed for transportation of church goers. Moreover, 70% of the cars so used were found to average 50 or more round trips to church during a year."

Night driving *is* dangerous driving as a graph made by the class plainly indicates. The problem has received considerable study from Safety officials and it is generally conceded that the main cause for night fatalities is poorly illuminated highways and the failure of the driver to appreciate new problems arising from impaired visibility.

Let us go back to the question of speed. One of my boys, Tulio, rather skeptically presented me, one day, with this question taken from Fun with Facts.

"Do you know that you can drive in darkness, even with your headlights turned on?"

I referred him to information on night driving in the New Jersey Manual for Drivers. He found there that "Average headlights provide safe vision for only about 150 ft. This," the statement continues, "is about the stopping distance at 40 miles an hour under *average* conditions." Tulio verified this statement by the stopping distance chart and announced his discovery to the class. So we learned *why* 40 M.P.H. is the maximum safe driving speed at night.

The glare of *approaching* headlights, we find, may prove just as dangerous as overdriving you own lights. The pupils of the eye, having contracted rapidly for protection from the glare, widen again very slowly. It sometimes takes a full minute for them to become normal. At 30 M.P.H. the driver has traveled 60×44 ft. or 2640 ft.—$\frac{1}{2}$ mi. in near darkness; at 40 M.P.H. he travels 60×58.7 ft. or 3522 ft. This we figure is about $\frac{2}{3}$ of a mile. Much can happen in that time.

Other similar studies stress the present dangers of night travel and we begin to appreciate why the driver, during these hours of darkness, should "proceed with caution." Eventually, we are told, highway engineers hope to illuminate all express highways thus contributing much to the reduction of this traffic hazard.

One of the boys, whose father is a road construction contractor, has been particularly interested in the simple material I have to offer him on highway engineering. He has prepared and presented to us at this time very interesting information on the meaning of road signs and markers; he has explained improvements and enlargement of old highways under way; he has demonstrated drawings of new super-highways which will provide all right hand turns and which, by overhead and underpasses, will do away with all crossing of highway traffic. Engineers hope, some time, he tells us, to provide pedestrian paths along all highways; pedestrian underpasses in the towns and cities. Radburn, New Jersey, a model community, built by the City Housing Corporation, already has such an arrangement.

Until such time as we have these accident-proof highways; or until the driver *can* or *will* do his part in reducing the every increasing highway toll, automobile insurance is most invaluable as a protection against financial loss. It is not my plan that a thorough understanding of automobile insurance be acquired by these children—I believe it is beyond them at this time. The importance of carrying cer-

tain kinds the types of protection afforded, the average cost, and the special rates to car owners who have avoided accidents, I do like to make clear.

We conclude our Safety unit by an analysis of traffic situations in Westfield. From information furnished by the local police department, we make spot maps showing danger areas. We make diagrammatical analyses of incorrect driving resulting in accidents, especially those occurring in these danger areas. We demonstrate by other drawings the correct procedures which should have been followed. We are quite pleased to note that Westfield's accident record has improved during the past year, and guarantee that we shall contribute to its further improvement.

Our *Know Your Automobile* study would not be complete without an inquiry into the costs of owning and operating a car. One father sent word to me, "If only these youngsters could be brought to realize the real cost of operating an automobile, and then be required to pay a certain part of this cost, perhaps they would be less eager to drive dad's car so much." Be that as it may, this information is vastly interesting when carefully worked out, and affords an activity in which the children eagerly participate. It is understood, at the outset, that the figures which we obtained are approximate. Costs of operating automobiles will vary not only with the make of the car, but with the care and use given it by the driver. The important thing is to discover the factors making up this expense and relatively how great that expense will be.

First, the children make up their own lists of things to be paid for in owning the automobile. These are compared and criticized. Some one offers the suggestion that a car costs more as it becomes older. From this exchange of ideas, we work out a table.

Then we decide to make a class study of some particular automobile—the children choose the Chevrolet—filling in the table as information is obtained and studied. Some facts we obtained from the fathers; other facts came from the dealers, service station and garage employees and directors. The children wanted to know these things and did their best to find them out.

In setting out for information *this year*, we were interested in learning how the defense program would affect availability and costs. We made many inquiries, read extensively in magazines, newspapers, automobile and petroleum periodicals. We found conditions in such a state of flux that very little *definite* information was available. Prices, too, were most uncertain; even during our period of study, price changes occurred which made it necessary for us to do some of our work over again.

To begin with, dealers estimate that the average car is driven 10,000 miles a year. This agrees with the facts from the fathers, so we accept them and fill in the information. Much discussion follows concerning mileage per gallon of gasoline and some very constructive comments are made concerning factors influencing gasoline consumption. We decide upon 22 miles as an average for the Chevrolet and determine 455 gallons as yearly consumption of gasoline. We are reminded that as the car gets older it burns more gas and uses more oil, so we act upon this suggestion in filling in our table. At the current average price per gallon—17.5¢—we figure gasoline costs for each of the first two years $80.00, increasing to $83.00 and $88.00 per year during the next three years.

We are advised that oil should be changed six times a year when the car is new. The oil capacity is 5 qts. *Figuring* 30 qts. of oil @ 35¢ we find the oil cost to be about $10.00 the first year. We note the increased consumption and cost in subsequent years.

From our study of several days back, we recall an insurance cost of $52.00 a year as sufficient to give us the protection we need.

Good tires should last about 20,000–25,000 miles, so we set aside $50.00 each of the third and fifth years for new tires. We are also advised to allow $9.00 for new batteries for each of these two years.

Inquiries reveal that license fees include $3.00 for driver's license and 40¢ per horsepower (A.M.A. rating—Auto Manufacturers Association) for car license. This makes our fees for the Chevrolet $15.00.

Now, automobiles must be serviced and repairs made when necessary. We have had great difficulty this year in getting estimates from various dealers to agree. We finally "struck an average" of figures submitted to arrive at our item of repairs cost.

Let us assume, this time, that we have paid cash for this car—$920.00 when it was new. The annual depreciation becomes a part of operating expense. Present estimates on trade in values are generous—so we have relatively low depreciation costs.

We finally total these costs—year by year. Most men are paid once a week, or once a month. So we are interested in determining how much of this wage or salary would have to be set aside each month to maintain the automobile. We find it runs from $24.00 to $32.00 a month. Quite a bit, isn't it when we realize that the average American pays for this and for all the other necessities of life for his family out of a monthly wage of $125.00 or less.

It will be recalled that during this time, in the English and Science classes, these children have been making intensive studies about specific cars of their own choice—frequently the family car. They have been organizing this material and putting it into booklets. To all this, they now add a page of estimated costs—such a study as we have just made of the Chevrolet.

In the meantime, certain boys have been arguing about the merits of their favorite cars and have urged their teachers to take sides. Finally, this proposition was made to them "You boys may stage an automobile show here on a date you may agree upon. Bring whatever materials you have or can get which will help you display the merits of your car. We shall come to your show and listen to your demonstrations. Whatever boy gives—in the opinion of us all—the finest and most convincing talk, that boy we shall declare the winner of this argument. Is it a go?"

And what a show we had! Attractive posters and cardboard models furnished by local dealers, simple electric signs made by the boys, effective drawings of special engineering features whereby these new devices could be more effectively explained! All talks were interesting and convincing and showed very creditable use of English expression—but the Buick was the winner.

Not to be outdone, the girls staged a Safety Show, organizing and displaying most attractively the *materials* used and results of work done in our Safety unit. To tell you the truth, their exhibit was much more persuasive in its appeal than the boys' show. And we were very glad. It was just the climax we had hoped for.

And now, let us evaluate this activity. Has it been worth while? Has it accomplished what we had hoped it would? How materially have we added to the sum total of these children's education?

First of all, I believe we have made them *safety* conscious. In so doing, I hope we have inculcated in them something of civic responsibility—that we have given them an appreciation not only of the rights of free citizens, but also of the obligations of those citizens to society in the promotion of general welfare and happiness.

Through the careful study of costs, we have attempted to develop something of a sense of values based on the conclusions of both *science* and experience. Through the recognition of the importance of super-highways, we see money well spent when invested to protect human life.

By providing meaningful experiences

which have appealed to the special aptitudes and interests of those boys especially keen about mechanical science and its applications, perhaps we have begun *ever so little* to educate toward self-realization and successful service.

Through all this there have been afforded many experiences in the use of fundamental skills, gathering and judging factual material and determining its use—experiences which *should* have direct transfer value to life situations.

If the effectiveness of education is measured by the translation of its ideals into experiences leading to specific goals, are not studies such as this worth while—real contributions to the social and intellectual development of our young people!

The Pre-Induction Training Program

demands more emphasis on mathematics. Speed up your High School courses with these aids:

ALGEBRA IN EASY STEPS, by Edwin I. Stein

Save time for teacher and pupil by these special features:
Individualized assignments
Automatic testing program
Abundance of practice material
New approach to word-problem solving

PROBLEMS IN JUNIOR MATHEMATICS, by Gilmartin, Kentopp, and Dundon

Develop your pupils' reasoning power with these more than 2000 concrete problems—about two-thirds in arithmetic, the rest in algebra, trigonometry, plane geometry.

DIAGRAMS IN THREE DIMENSIONS FOR SOLID GEOMETRY, by Breslich

With colorscope. Help pupils to visualize spatial relationships by means of these boxed sheets of 15 diagrams in three dimensions, on 8 x 9 heavy stock.

72 Fifth Ave. NEWSON & COMPANY New York, N.Y.

Publishers of Better Schoolbooks—*The Results Show the Difference*

New Objectives for Ninth Grade Mathematics

By WILLIAM WILLITS
Southern Junior High School, Reading, Pa.

I HAVE an interested eye on two of our present ninth grade sections. They were sectioned as average pupils on the basis of their past school performance. The two groups total 87 pupils. Intelligence test scores range from 122 to 84, with a median of 106. Scores on the Metropolitan Advanced Arithmetic test, given the sixth month of eighth grade, show a range of 9.8 to 6.3, with a median of 8.6. There are no honor roll students in these two groups; report marks of *C* quality are the rule; failing marks are not unknown. The mathematics record reveals that one pupil out of six has failed the subject in seventh or eighth grade. One out of six likes mathematics best of their study subjects, one out of four likes mathematics least, and the rest, about 7 out of 12, are non-committal about it—they neither like nor dislike the subject. If they had their choice in the matter of taking mathematics in senior high school—and they will have that choice—36 say they would take more mathematics, 51 say they would not. One-fourth of these pupils say already that they will not complete senior high school or are undecided.

We have here a group picture of the average ninth grade pupil. This is the group about which the problem of ninth grade mathematics revolves. These are not the scientists, the technicians, the engineers of the next generation; the need for a working knowledge of specialized mathematics fields, studied in rigorous sequential order, is not theirs. These are not the pupils who, by and large, can achieve the abstractions of secondary mathematics; the ability is not theirs. These are not the pupils who share our respect for the power of mathematics, our admiration of its logical organization; a necessary degree of interest and appreciation is not theirs. These are the pupils who will find their vocational places in store and office, shop and mill; their mathematics needs will be simple and specific. These are the pupils who need to develop a relatively few mathematical skills. They need to develop habits and patterns of thinking about number and quantity.

Whether we like it or not, ninth grade mathematics is a terminal course for the great majority of this group. Probably one pupil in five will select a senior high school curriculum in which mathematics is required, or will elect mathematics because he wants more of it. Whatever our offering in the ninth grade may be, then, it ought to be basically a terminal course, without, however, penalizing the pupil who needs or wants mathematics in the following years.

These statements give direction to our new objectives. We need for ninth grade mathematics a terminal course—with an open end—a course which teaches rigorously some few skills, old and new, and which is tied together throughout by a conscious development of a thinking pattern.

Algebra does not meet these requirements. A year's work in algebra is incomplete in itself; its abstractness leads all too often to a rote learning which has no functional value for the learner. Let's leave algebra for the superior pupil who will find essential use for it. The general mathematics appearing in texts as "Living," "Everyday," "Social," mathematics, is inadequate. To the pupil who has made a fairly good job of learning seventh and eighth grade mathematics, such a course must appear as "some more of the same."

Mathematics is an important part of the school curriculum because there is need for a knowledge of how quantities, represented by number, may be handled in order that problems may be solved. I

believe we lose sight of this simple fact in the mathematics teaching beyond the eighth grade. Our energies are spent in teaching a wonderfully organized body of subject matter. The superior pupil accepts this teaching, partly because he finds academic satisfaction in successful learning, and partly because he sees meaning in the complexity and abstractness of the subject matter. But the average pupil asks "What's the use of learning algebra?" and we are hard put for an effective answer.

Ninth grade is the place in which the mathematics learning of the average pupil can be consolidated, the consolidating idea being *problem solving*. We do not really teach pupils to solve problems in our seventh and eight grade mathematics. We are too busy teaching the fundamental operations—the tools of problem solving. The nearest we come to problem solving is to clothe these fundamentals with words, giving meaning to the manipulated numbers, and a name to the result. Problem solving below the ninth grade is a two phase cycle of principle learning—principle application. And this is probably a sufficient concept for the learners of these grades.

If we set the problem solving process as the central, unifying idea of ninth grade mathematics for the average pupil, we need to analyze the process. Is there a step-by-step technique which can be brought into high relief and which can be consciously taught and learned as a pattern? The two committee reports issued recently—that of the Joint Commission and that of the Progressive Education Association—point toward such a technique. Drawing from these sources, I propose the following steps in the process of teaching pupils a technique of problem solving:

Putting first things first, pupils should learn to recognize a problem situation. This idea is so elementary that we overlook it entirely in our teaching. Our practice is to create narrow problem situations, and scramble at once for a numerical result. For example, in teaching fire insurance in eighth grade, do we have pupils recognize the fact that the *idea* of fire insurance is itself a solution to a problem, the problem of how a property owner, through association with other property owners, shares fire losses in order that large individual losses are circumvented? Last summer the Bell Telephone Company ran in an advertisement a graph showing the distribution of long distance telephone calls through an average day. There was a peak load at 10:30 A.M., another at 2, and a third at 7 P.M. I reproduced the graph in a test, and asked the two "average" sections whether the telephone company, in publishing the graph, was seeking a solution to a problem. Thirty-seven per cent of them recognized no problem; they thought the graph was published because the company felt people might be interested in knowing when most long distance calls are made. Sixty-three per cent recognized a problem, but 12% thought the idea was to have more people make calls at the three peaks. Only 40% of the 87 pupils recognized the true problem: that the company wished to scatter the long distance load and avoid the peaks.

Second, pupils should be given the experience of collecting the data necessary for understanding and solving a problem. The textbook has been alpha and omega in mathematics teaching. Therein are found all the necessary skills, all the necessary data neatly set forth ready for use. It is time saving to have all data incorporated in the problem, time consuming—but a valuable experience—to have pupils search it out. Rotating pupil committees may function in the capacity of a Steering Committee to gather and present the data for a particular problem.

The library becomes a place useful to the mathematics department; pamphlets, other textbooks, magazines, even the daily newspaper, become sources of problem material and data. Thus for a problem on the Wage-Hour law our committee went to a government pamphlet on the subject; for a problem on "When Does Water

Boil?" it used a pamphlet on Yellowstone National Park and a Saturday Evening Post article; for a problem on "What are the Six Simple Machines?", a science text and an encyclopedia. It is surprising how quickly pupils learn that a problem must be *real* if you can find information on it outside of a mathematics book.

Third, pupils should have experience in selecting the best method of representing data. Does the problem yield a precise word statement? Can it be expressed as a formula? Does the formula yield a table of values for the variables involved? Can a graph be drawn? Of what value is the graph? What does the shape and slope tell about how the variables are related? Is an equation involved? What kind of equation—linear, radical, a system, quadratic? How are formula and equation alike? How different?

Here again our problem solving technique is not economical of time. It is less time consuming to teach a unit of subject matter, organized on a things-to-be-learned-and-later-applied basis. The problem solving technique reverses the process. A particular problem may call for fractional equations in its solution, or negative numbers in the drawing of a graph. If these are new experiences, they are taught for immediate use as part of the problem. Problem solving means new-learning-when-the-situation-demands. New learning is at once meaningful.

Lastly, pupils should be taught that there is a possibility of making a mathematical generalization in many problems, that generalizing ideas are a part of a problem's solution. For example, our problem on the Wage-Hour law yielded a formula of the type $y=ax$. A problem on comparing Fahrenheit and Centigrade temperatures yielded a formula of the type $y=ax+b$. And the problem on "When Does Water Boil?" gave a formula of the type $y=a-bx$. Examination of the graph in each case led to generalizations as to the relation between y and x, the significance of the a and the b, and the effect of the plus and minus signs.

By this time there are undoubtedly some serious questions to be raised. One may ask, "Just what is a problem? 'Problem' must be defined before problem solving can be taught." Another, "Can a course of study be prepared, consisting of a series of problems, which will have a satisfactory degree of coherence in its mathematical content?"

For our purposes, a problem situation must be real, and it should involve number and quantity to an extent making it worthy of study. It is a good problem if it can be stated concisely in question form, challenging the pupils' effort and aiding at once in recognizing the problem to be solved. Data for solution should be easily at hand—the average ninth grade pupil is neither expert in research nor in reading. Thus, "How Does the Federal Wage-Hour Law Work?" is a good problem, as are "What Should I Know about the Income Tax?", and "How Does Boiling Water Cool?"

Pupils should be led to recognize certain problem *types*, and be given experience with a variety of them. A valuable feature of the problem solving idea is that not all problems yield precisely accurate results. If the varying elements concerned in a problem—let's call them by their mathematical name—the independent variables—are all known and can be taken precisely into account, the problem's solution is accurate. Under the Wage-Hour law, a man's wages can be figured to the cent. Time worked is the only variable, and the law states specifically what constitutes "time" and "overtime." Not so, however, with the Income Tax Law. Provision is made by law for certain constants and variables, but at least one variable—the exemption for dependents—is made a constant for obvious reasons. And who among us is clever enough to know exactly what "Taxes Paid" to deduct? Again, the formula used in handling a jackscrew might show a 40 pound force lifting a ten ton weight. But the formula omits the

friction variable, which accounts for a loss of 50% or more.

So pupils should learn that problem results are accurate as the variables are known and accurately used, and that results are approximate to the extent that the variables are unknown or inaccurately considered. Somewhere also among our problems there ought to be a type involving subjective elements, personal predilections. A problem of this type might be "What Should I Know about Consumer Credit?" for which a Pollak Foundation pamphlet is an excellent reference. The State of Pennsylvania is faced with a problem of this type in reapportioning its congressional districts to reduce the number from 34 to 33. It is well for pupils to know that mathematical knowledge may temper desires and prejudices to a more rational solution.

There is a fourth type of problem which ought to come up for occasional consideration. Many a problem in everyday life involves no computation; there may be no number involved. The solution is a matter of judgment on the basis of available evidence. A roadside sign blazons, "Support the Townsend Plan and it will support you." What is the evidence that it will or will not? In the difficult years immediately ahead, cool judgment, clear thinking, correct conclusions will be an important part of national morale. Can mathematics play a part in the development of these habits? It can if we extend our teaching beyond the computational.

To weld these ideas into a course of study requires research and experiment. Research is necessary to determine the mathematical content which selected problem situations should cover. The two committee reports already referred to are valuable in this connection. Experiment is needed to test the effectiveness of the selected problem situations. If the textbook chapter or unit is replaced by the problem as a teaching unit, there is no reason why the same degree of coherence can not be achieved. One problem can lead as naturally to another as can one unit to another.

Problems will of course vary in length. Our problem on "What Are the Six Simple Machines?" took a month to cover, and we emerged with a knowledge of the lever, the wheel and axle, the cord and pulley, gear and pulley speeds, the inclined plane, the wedge, the screw, inverse proportion, and equations of the types $ax=b$ and $x/a=b/c$. Other problems studied covered periods varying from a day to two weeks.

Tests on problems make effective use of true-false, completion, and other new type questions, as well as the usual questions involving mathematical skills, for we are testing understanding of the problem, not merely the mathematics involved.

A course of study built on the problem solving concept can be dynamic. A core of problems will provide common experiences for all, and include the skills the course intends to cover. Additional problems may be planned for independent solution by individual pupils. Suggestions of problems from the pupils themselves should be invited and made a part of the year's work. Problems of current interest may find place one year and be replaced the next.

In the last analysis, what we want is *effective* learning—learning affecting the behavior patterns of the individual. We can accomplish something in this direction by reorganization and re-emphasis, by enlarging the pattern of our teaching. The learning of scattered skills—the partial learnings—may be forgotten, but the larger pattern of problem solving will endure.

Last June a former pupil, about to be mustered out of the army, stopped in at school. He read the exercises in algebra on the board haltingly, and said, "I used to know how to do those things." And I thought, "Losing that ability, did you lose everythong you learned?" Probably. Some years ago a former pupil, then already through college, was brutally frank. "I studied mathematics with you," he

said, "but I don't remember a thing you taught me." He was born some years too soon. He might have said, "I don't remember the mathematics you taught, but I do remember that we dug out material in the library and other books. We didn't just get answers, we solved problems. Once in a while I still dig out things that way."

Regional Meeting of the National Council of Teachers of Mathematics

Horace Mann School Auditorium, Teachers College, Columbia
December 31, 1942

MATHEMATICS FOR NATIONAL SERVICE

9:30 Letters from the Army and Navy

9:45 Mathematics and the National Crisis
William Betz

10:30 A Proposal for Mathematical Education in the Secondary Schools of the United States
William D. Reeve

11:30 Open discussion. (Leaders to be announced)
What are our schools doing to meet the national crisis?
In the proposed "air conditioning" program, is mathematics to become merely a tool for solving aeronautical problems?

12:30 Luncheon

2:00 Sixth Corps Area Conference on Pre-Induction Courses.
Edwin W. Schreiber

2:30 Gearing Mathematics to the War Effort.
Raleigh Schorling

3:30 Open Discussion (Leaders to be announced)

How can teachers get the training necessary for a satisfactory presentation of industrial, military, and aeronautical problems?

How can we use the present crisis to emphasize the absolute necessity of fundamental training in arithmetic in our elementary schools and a twelve year course in mathematics?

A 75¢ luncheon will be served for those who wish to attend on December 31, at the Teachers College Cafeteria. Reservations for this luncheon should be sent to W. D. Reeve, Teachers College, 525 W. 120 St., before December 25.

Vitalizing Mathematics through Classroom Atmosphere

By Ida Mae Heard
Marshall Junior High School, Marshall, Texas

Have you tried "face lifting" on your mathematics classroom? We did and with most gratifying results.

Two years ago my room was as bare as Mother Hubbard's cupboard—just a typical schoolroom. My pupils and I decided to individualize it. We wanted a room with personality—one that reflected some of the atmosphere of mathematics, a room more conducive to study.

Financing the project was our main problem. I sought school funds for such a project, but found, fortunately, that none were available. I say *fortunately* because I believe we would not have accomplished nearly so much had the money been readily accessible. In other words, I believe we have done *more* by having had *less;* the pupil participation in planning our classroom has probably been greater and the results more satisfying.

The children suggested and promoted various schemes for raising money to further our plans. First, we had a popcorn ball sale at school during the lunch period; then, just before Thanksgiving, we had a fruit sale; and later, each child donated 25¢.

With this money we purchased some pictures for our walls and our files. In choosing our pictures we were careful to see that they illustrated various principles of mathematics. Persons interested in our project have donated other appropriate pictures. To provide variety in wall scenery we keep a reserve of pictures that are changed at intervals before the pupils tire of them.

Various departments of our school offered us their wholehearted cooperation in our project. Activities were correlated wherever possible. The home economics students of our room designed and made the window draperies. The boys who took shop framed our bulletin board, built book shelves and ruled off a graph board. This was an excellent way to correlate their mathematics with their shop work.

During our study of line symmetry my pupils became interested in the butterfly as an example in nature. Numerous butterflies were brought to class and we decided to make a butterfly collection. A local cabinet maker constructed a case and the Science Department cooperated by helping the students mount their collection.

When we decided to have some pot plants in our room, a committee visited a local pottery to price some flower pots. They were able to save by purchasing "seconds"—pots with slight imperfections. They procured the 25¢ sizes for 10¢. Most of our plants were donated. They were put in different colored pots and placed in our sunny east windows. These made our room spectacularly cheerful. We have a red geranium and a red pepper plant in white pots; pink begonias in blue pots; ferns in light green pots; and sansevierias in brown pots. These plants not only make our room more attractive but offer a number of good examples of mathematical forms and principles in nature: the geranium has a symmetrical leaf; the red pods on the pepper plant are spheres; and one of the begonias has a leaf with a perfect spiral curve.

The Art Department helped with the arrangement of our room. We decided to let each wall present a unit. On the north wall, which is the back of our room, we have pictures that show the application of mathematics in nature. Our butterfly case hangs over a supply cabinet in the center of the wall space. On either side of the

butterfly collection hangs a water color done for us by the Art Department. One is a picture of dogwood, a flower indigenous to East Texas. My students have found great interest in the many legends woven about the dogwood, particularly the one claiming that the dogwood tree furnished the timber for the cross upon which Jesus was hung. The blossoms are in the form of a cross, two long petals and two short petals, and at the outer edge of each petal are nail prints, brown with rust and stained with blood. In the center is an image of a crown of thorns. The dogwood furnishes us a beautiful example of point symmetry. It can also be used to bring out the properties of quadrilaterals.

A painting of yellow jasmine is to the right of the butterflies. It also furnishes us an example of another mathematical principle: the lines joining the outstanding points of the petals form a pentagon. This suggests the poet's observation:

> Why Nature loves the number five,
> And why the star-form she repeats.

One may find in our room many more examples of mathematical principles exhibited in nature. We have a sweet potato vine on top of the supply cabinet. Its heart-shaped leaves illustrate line symmetry. In our supply cabinet there are several objects exemplifying different forms: a starfish that has a pentagonal form and a piece of honeycomb that contains hexagonal cells; a bird's nest that is a hemisphere and a collection of cone-shaped shells. There are many other examples of mathematics in nature to be found in pictures we have on file. One can not fail to appreciate the most wonderful of all the applications of mathematics—those to be found in nature.

Our east wall, with the four big windows, is used to show the application of mathematics in art and design. The window draperies have a tan background, with parallel lines of brown, yellow and blue. They serve as a frame for our colored print of Raphael's "Madonna of the Chair." This painting which hangs between the two groups of windows was chosen because it shows the use of the curved line in art.

Since I am Guardian of a group of Camp Fire Girls, I have their charter in my room. It hangs over our filing cabinet and serves a dual purpose—a club charter and an example of mathematical figures in design. The imaginative symbols of the American Indian, appearing in the border of the charter, represent many mathematical figures: the triangle, rectangle, square, hexagon, parallel lines, broken lines, and intersecting lines.

On the front, or south wall, we have pictures revealing facts in the history of mathematics. A large picture of the Tree of Knowledge hangs just back of my desk. Mathematics is placed at the base of the tree because of its importance to all scientific knowledge. On either side of this picture hang panels of three oil tinted pictures, each depicting an episode connected with some standard unit of measurement. These are very colorful and certainly help to add a zestful atmosphere to the classroom. My chair has a cushion appropriate for a mathematics room since the design is of squares and triangles.

To the right of my desk one section of the blackboard is ruled off with light green enamel for a graph board. The board is ruled in one inch squares, with every fifth line being heavy just like the graph paper the pupils use. This board has been invaluable in teaching graphs and the meaning of perimeter and areas. To the right are the book shelves that were made by the boys. On the top shelf are models of Chinese and Japanese abacuses and models of solid figures which the boys have made. We have a variety of sundial models that were made by the General Mathematics students when they completed their unit of work on "Fundamental Constructions." On a second shelf there is a representative collection of textbooks and reference books for the pupils and teacher. On the lower shelf is a general

collection of booklets. Two of the most interesting of these contain jokes and cartoons of a mathematical nature that were collected and compiled by my students.

The west wall has three large sized photographs, all of which were gifts. A striking picture of the national Capitol is in the center. This picture was taken at night and the lighted dome stands out in direct relief against the dark branches of the trees laden with snow. The picture might well be called a study in black and white. This photograph can be used to teach the use of geometry in architecture. The dome is a hemisphere and the columns are cylinders. On either side of this picture are autographed photographs of two internationally known contemporary mathematicians. Below these pictures is a large bulletin board on which we have displayed the prize winning posters in a recent poster making contest. Defense stamps were given as prizes to the pupils making the three best ones. Of course the uses of the bulletin board are numerous. It permits the display of all types of illustrative material and can be used as an effective teaching device.

The pupils take personal pride in our room now. We feel that the results of the project more than justify the expenditure of time and money. Some of the benefits derived follow:

1. The children received valuable training in earning money.

2. They received training in spending money wisely.

3. They learned to cooperate in a common objective.

4. The correlation of mathematics with other departments was effective.

5. The project aroused the interest and cooperation of many members of the community.

6. We have a classroom that is more conducive to learning.

7. Mathematics has become alive and practical to many pupils for the first time.

8. There is the satisfaction of having carried a task through to completion and of leaving something for succeeding classes to enjoy.

9. We feel that our classroom is as distinctive as any other in the building and that it personifies mathematics.

The purpose of this article is to show how by supervised cooperation any mathematics classroom can have a "face lifting." No attempt has been made to include all the things that an enthusiastic teacher might do, but only to suggest some things one teacher has done. It is hoped that the material presented may inspire similar projects and results.

Bibliography

I. *Classroom Reference Pictures in Use*

"History of the Standard Units of Measurement." H. G. Ayre, Western State Teachers College, Macomb, Ill. Set of six pictures, 8″×10′. Glossy finish $2.00; hand-tinted, $5.00.

"Madonna of the Chair" by Raphael. Practical Drawing Co., Dallas, Tex. No. 557y (Imp.) 15″ diameter. $3.50 unframed.

"Tree of Knowledge." Museum of Science and Industry, Jackson Park, Chicago, Ill. In colors, 25″×38″. 25¢ for single copy; 20¢ for each additional copy.

II. *Reference Pictures in Files*

"A High-lighted History of Algebra." Scott, Foresman and Co., Dallas, Tex. Free. This chart can be displayed on the bulletin board. It pictures the slow development of Algebra from the simple equations of Ahmes, 3600 years ago, down to its modern form.

"Mathematics of the Automobile." Free. The Educational Service Department, Chevrolet Motors Bldg., Detroit, Mich.

F. A. Owen Pub. Co., Dansville, N. Y. A nice collection of full-color prints of masterpieces can be secured to correlate mathematics and art.

The Perry Picture Co., Malden, Mass. Excellent reproductions of famous buildings to use in correlating mathematics and architecture are listed. Illustrated catalogue, 15¢.

Practical Drawing Co. Dallas, Tex. Full-color prints of masterpieces.

Scripta Mathematica Portfolio of Portraits of Eminent Mathematicians. Scripta Mathematica, 610 W. 139 St., New York City.

III. *Other References*

Bedford, Fred L., "Planning the Mathematics Classroom." Reprint from *The School Executive,* Vol. 55, pp. 290–292, April, 1936. Free on request. Lafayette Instruments, Inc., 252 Lafayette St., New York City.

Cook, Mary Ruth, "Stimulating Interest in Mathematics by Creating a Mathematical Atmosphere," THE MATHEMATICS TEACHER, vol. 24, pp. 248–254, April, 1931.

Kee, Olive A., "A Mathematical Atmosphere," *National Council of Teachers of Mathematics, Third Yearbook,* pp. 268–276.

Mossman, Edith L., "A Mathematics Room That Speaks for Itself," *School Science and Mathematics,* Vol. 33, pp. 423–430, April, 1933.

Waters, Idells, "Vitalizing Geometry through Illustrative Material," THE MATHEMATICS TEACHER, Vol. 28, pp. 101–110. Feb., 1935.

Woodring, N. M. and Sandford, Vera, *Enriched Teaching of Mathematics in the High School.* New York: Bureau of Publications, Teachers College, Columbia University, 1938, pp. 104–112.

Just published

—the fundamentals of mathematics with industrial applications—

•

Wolfe, Mueller and Mullikin's

SIMPLIFIED INDUSTRIAL MATHEMATICS

Developed in the apprentice schools of the Ford Motor Company, this new book is designed to be used in all kinds of vocational, trade, industrial, and apprentice training programs. Geared to the comprehension of average students, it begins with a review of arithmetic fundamentals and progresses to precision instruments, symbols, formulas, ratio and proportion, tapers, percentage, geometry, trigonometry, screw threads, and gears. $2.00.

Write for further information.

McGRAW-HILL BOOK COMPANY, INC.
330 West 42nd Street New York

A Calendar of the Birthdays of Mathematicians[1]

By JAMES D. TELLER

College of Education, Ohio State University, Columbus, Ohio

Month	*Day*	*Year*	*Name*	*Birthplace*
Jan.	3	1777	Louis Poinsot	Paris, France
	5	1838	Camille Jordan	Lyons, France
	11	1865	Johannes Franz Hartmann	Erfurt, Germany
	11	1825	*William Spottiswoode	London, England
	14	1819	Sir James Cockle	Essex, England
	15	1850	Sonya Kovalevsky	Moscow, Russia
	20	1831	Edward John Routh	Quebec, Canada
	22	1592	Pierre Gassendi	Champtercier, France
	23	1693	Georg Bernard Bülffinger	Württemberg, Germany
	25	1736	*Joseph Louis Lagrange	Turin, France
	27	1832	Charles Lutwidge Dodgson[2]	Chesire, England
	28	1540	Ludolph Van Ceulen	Hildesheim, Holland
	28	1701	Charles Marie De La Condamine	Paris, France
Feb.	11	1839	Josiah Willard Gibbs	New Haven, Connecticut
	13	1805	Peter Gustav Lejeune Dirichlet	Düren, Germany
	14	1514	*Rheticus	Feldkirch, Germany
	15	1564	*Galileo Galilei	Pisa, Italy
	19	1473	*Nicolaus Copernicus	Thorn, Poland
	26	1786	Dominique Francois Jean Arago	Estagel, France
	27	1867	*Irving Fisher	Saugerties, New York
	28	1683	René Antoine Ferchault de Réaumur	La Rochelle, France
March	3	1845	Georg Cantor	St. Petersburg, Russia
	5	1512	*Gerardus Mercator	Rupelmonde, Flanders
	10	1748	*John Playfair	Benvie, Forfarshire, Scotland
	14	1879	*Albert Einstein	Ulm, Württemberg, Germany
	16	1787	*Georg Simon Olm	Erlangen, Germany
	17	1780	August Leopold Crelle	Wriezen, Germany
	20	1834	*Charles William Eliot	Boston, Massachusetts
	21	1768	Jean Baptiste Joseph Fourier	Auxerre, France
	26	1773	Nathaniel Bowditch[2]	Salem, Massachusetts
	28	1749	*Pierre Simon LaPlace	Beaumont en-Auge, Normandy, France
	31	1730	Étienne Bézout	Nemours, France
	31	1596	*René Descartes	La Haye, Touraine, France

[1] All data are taken from *The Encyclopaedia Britannica*, 14th edition, unless otherwise indicated. Such a calendar is intended primarily as an aid in humanizing the teaching of mathematics. It should prove valuable in suggesting appropriate bulletin board displays and exhibits. However, the calendar has definite limitations. The birthdays of many mathematicians are not known. Thus, Pythagoras, Plato, Hipparchus, Euclid, Archimedes, and many others are omitted from the calendar for this reason alone. Moreover, the calendar is not intended to be exhaustive but merely suggestive. Classroom teachers would render a service to each other if they would report the ways in which the calendar has been of use to them. Such suggestions are welcomed by the editor and author and will be assembled in a future article with due acknowledgement as to their sources.

[2] Data for this entry are taken from *The Encyclopedia Americana*, 1939 edition.

* The starred names have been used by the writer in various bulletin board projects, museum exhibits, and assembly programs during the past twelve years.

Month	*Day*	*Year*	*Name*	*Birthplace*
April	4	1809	Benjamin Pierce	Salem, Massachusetts
	10	1766	Sir John Leslie	Largo, Fifeshire, Scotland
	15	1707	*Leonhard Euler	Basel, Switzerland
	16	1823	Ferdinand Gotthold Eisenstein	Berlin, Germany
	25	1849	Felix Klein	Düsseldorf, Germany
	25	1857	Alexander Nikhailovich Liapunov	Russia
	28	1831	Peter Guthrie Tait	Dalkeith, Scotland
	29	1854	*Jules Henri Poincaré	Nancy, France
	30	1777	*Karl Friedrich Gauss	Brunswick, Germany
May	2	1601	Athanasius Kircher	Geisa, Fulda, Germany
	4	1733	Jean Charles Borda[2]	Dax, France
	4	1845	William Kingdon Clifford	Exeter, England
	5	1809	Frederick Augustus Porter Barnard	Sheffield, Massachusetts
	7 or 13	1713	Alexis Claude Clairault	Paris, France
	16	1718	Maria Gaetana Agnesi	Milan, Italy
	26	1667	*Abraham De Moivre	Vitry, Champagne, France
	28	1676	Jacapo Francèsco Riccati	Venice, Italy
June	6	1842	Henry Martyn Taylor	Bristol, England
	11	1743	Robert Hamilton	Edinburgh, Scotland
	16	1801	Julius Plücker	Elberfeld, Germany
	18	1858	Andrew Russell Forsyth	Glasgow, Scotland
	19	1623	*Blaise Pascal	Ferrand, Auvergne, France
	21	1781	Siméon Denis Poisson	Pithiviers, Loiret, France
	27	1835	Domenico Comparetti	Rome, Italy
	27	1806	Augustus De Morgan[2]	Madura, Madrus, India
	29	1840	Francis Huntington Snow	Fitchburg, Massachusetts
July	1	1646	*Gottfried Wilhelm Leibnitz	Leipzig, Germany
	1	1788	Jean Victor Poncelet	Metz, France
	2	1852	William Burnside	London, England
	10	1682	Roger Cotes	Leicestershire, England
	11	1857	Sir Joseph Larmor	Magheragall, Ireland
	13	1527	John Dee[2]	London, England
	14	1793	George Green	Nottingham, England
	17	1698	Pierre Louis Moreau De Maupertius	St. Malo, France
	22	1795	Gabriel Lamé	Tours, France
	27	1801	Sir George Biddell Airy	Alnwick, England
Aug.	3	1863	Alexei Nikolaievich Krylov	Alatyr, Simbirsk, Russia
	8	1602	Gilles Personne de Roberval	Roberval, France
	13	1842	Jean Gaston Darboux	Nîmes, France
	13	1819	Sir George Gabriel Stokes	Skreen, Ireland
	16	1802	Moritz Wilhelm Drobisch	Leipzig, Germany
	16	1845	Gabriel Lippman	Holerich, Luxembourg
	17	1601	*Pierre de Fermat	Beaumont-de-Limagne, France
	18	1685	Brook Taylor	Edmonton, Middlesex, England

Month	*Day*	*Year*	*Name*	*Birthplace*
Aug.	21	1789	Augustin Louis Cauchy	Paris, France
	21	1821	Arthur Cayley	Surrey, England
	23	1829	Moritz Cantor	Mannheim, Germany
	25	1802	Niels Henrik Abel	Findöe, Norway
	26	1728	Johann Heinrick Lambert	Mulhouse, Alsace
	28	1867	Maxime Bocher	Boston, Massachusetts
Sept.	5	1725	Jean Étienne Montucla	Lyons, France
	11	1877	*Sir James Hopwood Jeans	London, England
	11	1798	Franz Ernst Neumann	Joachimstal, Germany
	17	1826	*Georg Friedrich Bernhard Riemann	Breselenz, Hanover, Germany
	18	1752	*Adrien Marie Legendre	Paris, France
	21	1776	Christian Ludwig Ideler	Perleberg, Germany
	24	1501	*Girolamo Cardan[2]	Pavia, Italy
	29	1803	Jacques Charles Francois Sturm	Geneva, Switzerland
Oct.	6	1831	Julius Wilhelm Richard Dedekind	Brunswick, Germany
	7	1875	Raymond Clare Archibald	Colchester, N.S., Canada
	11	1758	Heinrich Wilhelm Matthias Olbers	Arbergen, Germany
	15	1608	*Evangelista Torricelli	Faenza, Italy
	26	1877	Max Mason	Madison, Wisconsin
	31	1815	Karl Weierstrauss	Ostenfelde, Germany
Nov.	2	1815	George Boole	Lincoln, England
	2	1793	Nicolai Ivanovich Lobachevski	Makariev, Nizhni-Novgorod, Russia
	2	1826	Henry John Stephen Smith	Dublin, Ireland
	12	1746	*Jacques Alexandre César Charles	Loiret, France
	15	1839	Michel Chasles	Epernon, France
	16	1835	Eugenio Beltrami[2]	Cremona, Italy
	17	1790	August Ferdinand Möbius	Schulpforta, Germany
	23	1616	John Wallis	Ashford, Kent, England
	29	1803	*Christian Johann Doppler	Salzburg, Austria
	29	1847	Sir Alfred George Greenhill	London, England
Dec.	10	1804	Karl Gustav Jacob Jacobi	Potsdam, Germany
	17	1842	Marius Sophus Lie	Nordfjordeif, Norway
	21	1542	Thomas Allen	Staffordshire, England
	22	1765	Johann Friedrich Pfaff	Stuttgart, Germany
	25	1642	*Sir Isaac Newton	Woolsthorpe, England
	26	1792	Charles Babbage[2]	Devonshire, England
	26	1796	Johann Christian Poppendorff	Hamburg, Germany
	27	1571	*Johann Kepler	Weil, Württemburg, Germany
	31	1864	Robert Grant Aitkin	Jackson, California

What the Schools Should Do to Further Defense*

By E. A. Bond
Western Washington College of Education, Bellingham, Wash.

My topic for this morning's program was submitted before the seventh of December. Yet the theme is far more important now that it was before that date. At the outset, I want to say that the schools should carry on as far as possible as they have been doing but with a stronger drive. Our work should be augmented and our efforts should be pointed more nearly towards our preparedness problems. Our national need is great. Indeed, America faces a number of supreme needs to meet which will take the combined powers of all of our people. Of these needs I shall speak of some which I consider to be paramount. In doing so, I purpose to show the ways that our schools can contribute towards supplying them.

First, we need greater precision of thinking and more accurate work than our people have ever known. The efforts of each one of us from the least to the greatest must be greatly increased in quantity and much improved in quality. We need to dedicate all our time, powers and property to the task of maintaining and improving the splendid culture we have built so that we will be in a position to enforce the inclusions of the ideals of individual security and freedom when world order replaces present chaos. Since the early days of the Republic, our country has never faced a task more exacting than the one now confronting it. Two months ago wer were very poorly prepared for self-guarding our country. But today we—a united America—have soberly and grimly decided to gird ourselves thoroughly and battle for our very existence and our rights as individuals.

But while the problems incident to this war are being met we should take a long view of the steps necessary for this preparation. We should be short-sighted in the extreme, if we allow the present exigency to curtail the program of the schools either through lack of income or of competent teaching personnel while we are arming ourselves and successfuly prosecuting this war. On the other hand, we should make sure that our youth shall have the chance to participate intelligently in protecting and bettering the heritage we shall leave them. Furthermore, our youth should be given the vision of what the America of tomorrow may and should become. Likewise, our youth should become aware of the part they will need to play and the price they will have to pay in order that the America they envision may be realized.

What America will be in the 50's and the 60's of this century will depend largely upon the intelligence, the moral stamina and the heroic stalwartness that our youth of today take to those years. Our national life will depend in no small measure upon what happens in our classrooms during the 40's. Hence our schools should be sustained in their effort to increase the precision of thinking and the accuracy of work of our youth. At the same time these qualities should be coupled with as sound an education as possible and still meet the supreme need of today. While the world is fast degenerating elsewhere we should make progress if we want our ideals to live.

Now, there is no better school subject than mathematics by means of which our pupils can develop the powers to do precise thinking and careful work. The principles of arithmetic and the laws and the language of algebra provide material on a concrete level which, when properly generalized, is par excellence for a basis of developing the ability to do careful think-

* Read at the Annual Meeting of The National Council of Teachers of Mathematics at San Francisco, February, 1942.

ing—the kind of thinking needed today in our all-out program of winning the war. It is also the kind of thinking that will be needed when the war is over or at any other time for that matter.

Again, the application of the principles and laws to the quantitative problems of life today afford chances for our pupils to practice doing careful work. Such work should not only be good but it should be just right. The attitudes of our pupils should become such that no pupil will be satisfied with an accomplishment that is inferior to the best he can do.

The methods and aims of the mathematical science should be cultivated to the extent that the applications to living problems will find an adequate place in instruction. The demand for skilled computational abilities is far beyond the supply of such workers. So is the demand for skilled technicians. It will tax the colleges for the next few years to find enough men and women of sufficient mathematical training for the extensive research problems now involved in our war efforts. Likewise it will tax the high schools greatly to provide the student personnel for those colleges. The call for men and women with broad cultural knowledge of mathematics is even now far above the supply.

This year I have had letters from our state University and from our State College asking for those of our graduates who have specialized in mathematics. Each of these schools offer limited scholarships to those wishing to further their training in this science.

Accordingly, it would seem to be our privilege and our duty to seek out those of our pupils who are most apt and make it possible for them to take four years of mathematics in the high school, and to induce them so to do. In the elementary school, and in the junior high school the foundation is laid for the future work of our pupils. Hence the work in these schools should point towards further mathematics as well as toward that which is needed in their present activities and which will be needed in the life activities to which our pupils will soon go. Yet at the same time the materials of instruction at the present should be closely associated with the demands for exact knowledge and precision of work that our country needs today.

The teachers of junior high school mathematics are in a better position than any other teachers to help pupils develop attitudes favorable to precision. Our materials are concrete. The work we are doing in interpreting and in making graphs-bar, divided-bar, and circle graphs is splendid material for this purpose. However, I feel that our efforts should be made as strong as possible to keep such work on a high plane of excellency. Moreover, the situations selected for teaching the understanding and making of graphs should be chosen so that they are socially important. Accordingly we teachers should strive to become skillful in finding and in improvising problem situations that are worthwhile at any time but are especially important now.

Each week, the United States Department of Labor, publishes the current wholesale prices of consumers goods with a comparison of current prices with those of a month ago and, also, with those of a year ago. This bulletin may be had for the asking. It contains up-to-date material that is ideal for teaching price index-numbers, price trends, ratios, graphs, etc.

The work we have been doing in direct measurements and in indirect measurements have long been used for practicing precise work. The scale drawings of triangles used in indirect measurements afford opportunities for pupils to develop the power to do careful work. Again, a study of the different ways that distances are expressed on maps and of reference-fractions under the pictures in the dictionary is another source of opportunities to practice precision. For example, a map of Asia, which I consult almost daily now,

has the reference-fraction 1/5,400,000. That means that one inch on this map stands for 5,400,000 inches on the continent of Asia. Hence the scale of miles on that map is 5,400,000 divided by 63,360 or 85.3 miles to the inch.

A study of the reliability of the results arrived at by computing gives insight into the care needed for precise work. We have not always been careful to round our results to agree with the precision of measurements of the data from which the result has been derived. Frequently a result is expressed in feet and hundredths of a foot when the measurements from which the result is found is given in even feet. Pupils should be led to know that no amount of precision of computing can improve the lack of precision in the measurement giving rise to the computing. Insight into the significance of the figures resulting from computing is a valuable aid in a precision program.

For example, the diameter of this circle was measured and recorded as 26.46 cm. Its length was computed and expressed with eight figures. But by establishing the limits within which the length of the circle lies it is seen that only the three left-hand figures are significant and that while the fourth figure from the left is the best guess it is not reliable:

26 46	26 455	26 465
3 1416	3 1416	3 1416
79 38	79 37	79 40
2 65	2 65	2 65
1 06	1 06	1 06
3	3	3
1	1	1
83.13	83.12	83.15

Hence, even though the diameter had been measured to the nearest hundreth of a centimeter, the best expression for the length of the circle is 83.1 cm. These three figures are significant and the next one is a nearest figure.

I spoke a moment ago of the need for our pupils to develop the attitude for, and the ability to do, accurate thinking. I spoke also of the splendid contribution that a proper study of algebra can make to both of these ends. But we must not forget that the subject of algebra is not now so popular as it was a generation ago. However, many competent thinkers feel that the loss of its popularity is due rather to bad teaching than to the nature of algebra. Yet in the ranks of administrators and of professional educators, there is a growing distrust of the usefulness of the delightful study of algebra. Unfortunately, their distrust is shared by too many teachers of ninth grade pupils. As a result of this distrust of the usefulness of algebra, this subject already has been wholly replaced in the ninth grade of many schools by an extension of the general mathematics now being taught to eighth grade pupils. The chief argument now being given for this change is the claim that too many pupils of the ninth grade have been unable to understand the work that has been presented to them.

Really, the trouble is neither with the pupils nor is it with algebra. The real trouble has been that too infrequently have the two been brought together. The nature of the work presented in algebra today does not differ much from what I learned in my boyhood when only the one best out of twenty studied the subject. But now every man's son and daughter enter the ninth grade. While our pupils today are better prepared on the average than were the boys and girls of their age in the 80's and 90's, the ones that enter the ninth grade are far less well prepared than were the group who went on to high school at that earlier time. So, of course, the teaching of algebra or of any other subject, for that matter, should be differently done and with a different purpose than in my time. Of course it should have been different all along in content, in methods and in purpose.

I want to show a type of algebra which will be just as rigorous and just as liberalizing, and at the same time far more

meaningful than the type of algebra against which the attack is now being made.

As I see the problem, algebra is generalized arithmetic with a few minor details added. The teaching for the most part should be from this point of view. The study should start where the pupils are, and the pupils should learn the language of algebra in a sufficiently concrete way. The meaning and the use of the material should have more attention than the mechanical phases of the subject.

Let me do some illustrating as we go along. Early in the elementary course, the pupils learn that $4+5$ is the same as $5+4$. From enough of these sums and reverses, they generalize that the sum of any two numbers is the same in either order. Now the language of algebra includes all such cases in the single sentence $(a+b)\equiv(b+a)$ where a stands for any number and b for any number which is usually different from a. Likewise $(a+b+c)=(b+a+c)=$ anyone of the four other ways in which the three numbers can be arranged. This one statement includes the 2187 different situations wherein the sum of three numbers each of which is less than ten can be arranged in any order. They soon learn that algebra is the language by means of which generalizations such as those under discussion can be concisely and conclusively stated.

Again, children in the elementary school learn how to add any two simple fractions. There are 4096 such situations in which both terms of each fraction is less than 10. They easily learn that the sum in every case is the sum of the cross products divided by the product of the denominators. The language of algebra expresses this fact like this:

$$\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}.$$

So this single formula gives the sum for each of the more than 4000 cases. Likewise

$$\frac{a}{b}-\frac{c}{d}=\frac{ad-bc}{bd} \text{ where } \frac{c}{d}<\frac{a}{b}$$

will take care of the more than 2000 cases of subtraction of fractions.

We have long taught addition and subtraction of fractions upon the low level of specificity, wading through innumerable specific examples without developing the generalizations which bind them into related groups and expressing each procedure in a single sentence in algebraic language.

In similar ways many other generalizations of arithmetic procedures can be used to give strength to arithmetical learning while the language of algebra is being learned and used. If such a program is followed the teaching of algebra will have more meaning to our pupils and at the same time their control of arithmetic will be maintained and extended.

A second major need of our country today is for a spirit of willingness to sacrifice on the part of all of our people. The common good is each ones best security. Every one of us should be glad to do all and give all to assure that our country will receive all the help it needs in the trying days ahead. No doubt there will be some injustices. Some groups of our people will suffer more than others. But whatever short rationing comes to any of us, if it can't be avoided should be borne with fortitude. For example, the tires on my car are bad. But since I can't get others, I shall make the most of the matter until the shortage has passed.

Now there is no better incentive for developing a spirit of willingness to sacrifice than an exact knowledge of the needs our country is experiencing. Here again the teacher of mathematics has an opportunity to do real service. Tables and graphs that show our productions, consumptions and our reserves and the trend of each of these are suitable for this purpose. Such work includes most of the mathematical objectives usually included in the junior high school mathematics courses.

Take for example, a study of our supply of wheat. Last year was in our country one

favorable for the production of wheat. We produced 628 million bushels. This amount is more than 100 million bushels in excess of the average for the preceding ten years. How shall we plan our production for next year with several possible contingencies facing us? E.g., next year may not be a favorable one for wheat production. Some of our wheat will be lost through enemy submarine activity. Our allies may be in urgent need of wheat in unpredictable ways. A study of such a problem is enticing to pupils and gives them insight into the uses of mathematics in studying social problems.

Here are some facts that should help to make our pupils loyal Americans:

1. *The population of the world is about 1900 million; that of the United States is about 130 million. So we have 6.8% of the world's population, yet our production of steel is 32% of the world's production.*
2. *We have 51% of the world's telephones. 28% of the world's railroad mileage, 62% of the the world's motor vehicles. We carry 54% of the world's air transportation. We produce 42% of the world's petroleum.*

These are but a few of the commodities that enrich living of which the United States produces far more than its share. We can build into the lives of our pupils real love for our country by showing what it has done for human betterment—that it has built the highest standard of living that has ever been enjoyed on this earth.

I shall mention a third need of our country today—an elusive one. It is for more resoluteness in facing the future—more assurance that we shall do our part to make this a better world in which to live and work. We need a better morale. The teachers of mathematics should join with others to overcome fear and lack of confidence in our ability to finish the job with charity for other peoples and without hatred for any.

Let me close this talk this morning with a quotation from Professor Hotelling of Columbia University with some comments of my own. I quote: "Mathematics, to my way of thinking, is the most general of all subjects. Everything else is more special. There is no school subject that has a richer profusion of applications. There is nothing that travels over the whole domain of human knowledge as does mathematics. There is no surer way to unlock all sorts of doors than mathematics. It won't get you all the way, but it will get you into places where you can't enter by any other method. It will supplement all other types of investigations and help get at profounder truths than will be possible without its aid." end of quotation. When one realizes that Professor Hotelling is not a mathematics teacher and that his equipment in mathematics is not of the first magnitude, the force of what he says must appear in an unbiased light. He is an economist rather than a mathematician.

Accordingly, I feel that for most students some type of mathematics would prove useful to them in each year of the high school both junior and senior. But the work should not be the same for all. At least two separate highways should be built through the high school beginning with grade nine if a dual or multiple program can be administered. One of these programs should give the best training possible in general mathematics that takes a broad view of the subject. The purpose of such a course should, I think, be rather to show the contributions of mathematics to our civilization and to the ways we are now living together than to attempt to develop skills, techniques and mathematical procedures. In other words the work should contribute to the student's understanding of the nature of mathematics and what it does and can do. Such a course could be made just as exacting as the abilities of the pupils will permit. It would contain less of mathematics and more about mathematics.

The second program should start with grade nine and permit the pupils to gain four years of rigorous mathematical study. It should be limited to students who have the ability and the inclination to do a lot of hard, painstaking work. However, I feel

that there should be the possibility of transferring from either to the other. This course would emphasize algebra as "the language of thinking." These last four words are not mine though the thought they express has long been in my mind. These words are those of Irving Fisher the economist, of Hotelling the sociologist, of Rogers the Chemist, and of Dobbs, the College President as well as of a host of mathematicians of the first order.

So let us carry on and do the best job we can do, realizing the importance of the subject we teach, and the contribution we can make by doing our work in a superior manner.

◆ THE ART OF TEACHING ◆

Teacher-Pupil Planning in Junior High School Mathematics

By Bjarne R. Ullsvik
University of Wisconsin High School, Madison, Wisconsin

The goal of all school activities, irrespective of the individual pupil or his level of maturity, should be directed toward the perpetuation and fostering of a democracy. The present international situation should place a premium on the democratic way of life, and our schools should rededicate their energies in developing pupils whose characteristics are conducive to a democratic way of life.

Teacher-pupil planning is an attempt to provide experiences for pupils that are more directed toward the pupil's conception of his own significance and belongingness in the classroom, and coupled with the pertinency of the school activity to mathematical objectives learning becomes more effective. It's a cooperative approach to curriculum making and a break from authoritative prescription.

The kind of problems that are real to junior high school pupils do not arise as problems in algebra, geometry, or trigonometry; nor are the solutions of these problems confined to any one of a set of fundamental mathematical concepts that occur in the solution of all such problems. Such inclusive concepts would be much more acceptable than the conventional categorization of secondary school mathematics. A Commission report of the Progressive Education Association, "Mathematics in General Education," has devised a set of such concepts.

Mathematics should provide the individual pupil with a method of thinking that will enable him to analyze the problem and proceed with its solution. The emphasis on manipulative techniques should be replaced by emphasis on mathematics as a way of thinking. Pupils must think through those problems that are real to them, if we are to emphasize mathematics as a way of thinking. Teacher-pupil planning will help provide this necessity.

Mathematics can make a contribution in the realm of "thinking" that should make the democratic way of living a perpetual heritage of American life.

EDITORIALS

Mathematics on the Home Front

IN RECENT issues THE MATHEMATICS TEACHER has been trying very hard to keep its readers properly informed as to the mathematics that is essential for the war effort. To this end articles by officers of the Army and Navy, as well as by educators, have been published; and we hope that they have been helpful. There seems to be a rather general feeling all around that the best plan is to teach the basic elements of a modern course in mathematics as well as possible and to show the pupils how to make applications that are real and important in fields like aviation, which are now coming to the fore. However, we think it would be a mistake to overlook the importance of mathematics on the home front, which will obviously be so necessary in the days of peace that lie ahead. Social and economic arithmetic, or the mathematics of the consumer, should not be overlooked. Here we have as great an opportunity to show the real worth of mathematics education to American citizens as we do in teaching solely war mathematics. The point is that we can and should do both.

From a position where mathematics was forced by events to fight for a place in most secondary schools, particularly after the eighth grade, the war needs have demonstrated that mathematics should be taught to many more pupils than formerly. Moreover, it is just as obvious to many of us that ordinary life needs of the individual also demand a greater knowledge of mathematics than our ancestors knew. A large part of this knowledge may be of an arithmetical nature, but it is highly important. We shall give some space in an early issue to some of the phases of mathematics on the home front.

These facts make it extremely important that those of us who are responsible for the organization and teaching of mathematics in the schools see to it that the course of study be modified to meet the needs of 1943 and not those of 1900. If this means a national cooperative effort to reorganize mathematics, so much the better.

W. D. R.

A Merry Christmas and a Happy New Year!

ONCE MORE THE MATHEMATICS TEACHER wishes its readers a Merry Christmas and a Happy New Year. We can extend such wishes this year with more confidence than in 1941, because we have reason to feel that, other things being equal, our prospects for peace are much brighter.

W. D. R.

Seventeenth Yearbook

The Seventeenth Yearbook of the National Council of Teachers of Mathematics, "A Source Book of Mathematical Applications," will be of great interest and help to those teachers who are looking for applications of mathematics that have a distinctly real value. Moreover, some of these applications are related to various phases of the war effort. A detailed description of the contents appears on the back outside cover of this issue. In order to be sure to secure a copy of this book, teachers should send in their orders at once to the Bureau of Publications, Teachers College, 525 West 120 Street, New York City.

W. D. R.

IN OTHER PERIODICALS

By Nathan Lazar

The Bronx High School of Science, New York City

Bagley, W. C., "Progressives Disavow Blame for Wartime Educational Weaknesses," *School and Society*, 55: 492–493, May 2, 1942.

Balbridge, C. C., "Number Work; Practical Lesson Plans and Games," *Grade Teacher*, 60: 26+, October, 1942.

Balbridge, C. C., "Your Arithmetic Class; Curriculum Plans and Teaching Helps," *Grade Teacher*, 60: 48+, October 1942.

Blesdoe, J. M., "Types of Mathematics," *Texas Outlook*, 26: 36–37, August 1942.

Brodie, S. S., "Navigation for High School Students," *High Points*, 24: 55–58, May 1942.

Curry, H. B., "Mathematical Teaching and National Defense," *School Review*, 50: 337–346, May 1942.

Denny, E. C., "How Good Is Your Arithmetic?" *Midland Schools*, 57: 44–45, October 1942.

Durrant, J. E., "Let's Cut the Suit to Fit the Cloth; Mathematics," *School* (Secondary edition) 31: 49–52, September 1942.

Hartung, M. L., "Increased Emphasis on Mathematical Training," *School Review* 50: 175–176, March, 1942.

Hawkins, G. E., "Selected References on Secondary-School Instruction; Mathematics," *School Review*, 50: 141–144. February 1942.

Hildebrand, J. H., "War and the Decimal Point," *School and Society*, 55: 543–547, May 16, 1942.

Lyons, C. A. and others, "Study of Mathematics and the National Defense," *Pittsburgh Schools*, 16: 228–233, May 1942.

Madill, F. E., "Make Your Own Clinometer," *School* (Secondary Edition), 30: 839, June 1942.

McCormack, J., "Interesting Calculation Methods," *Industrial Arts and Vocational Education*, 31: 309, September 1942.

Moore, L., "Mathematics in National Defense," *High Points*, 24: 5–7, June 1942.

Moore, L., "Mathematics Problems in Aviation," *High Points*, 24: 29–33, February 1942.

Norris, G. W., and Paterson, E. B., "Mathematics Mechanized Warfare's Sharpest Weapon," *Baltimore Bulletin of Education*, 19: 193–197, April 1942.

Orleans, J. B., "Evaluation in Secondary Mathematics," *Secondary Education*, 10: 314–317, February 1942.

Roberts, O., "Arithmetic Needed by Industrial Employees," *Texas Outlook*, 26: 27–28, August 1942.

Weber, V. L., "Teaching Arithmetic through Defense Problems," *Education for Victory*, 1: 6, July 15, 1942.

Willey, R. D., "Functional Arithmetic, 1893–1940; A Review of a Typical Theoretical Discussion and the Theory to which it Has Led," *Journal of Educational Psychology*, 33: 105–117, February 1942.

Merry Christmas! 1942 Style

Buy War Bonds and Savings Stamps and thus insure that there will be a Happy New Year as well.

215 ***competitive skills games, with scoring, for teaching the individual skills of 8 sports:***

Teaching Athletic Skills

IN PHYSICAL EDUCATION

By **HENRY C. CRAINE**

Foreword by Dr. Jay B. Nash

1. This book takes the *individual skills* of 8 different sports, and makes exciting competitive games, *with scoring,* of them. An average of 27 graduated, coordinated skills games for each sport! The play way, with no monotonous drill, pupils improve in each skill of a sport, and learn to play the sport better. Here's a program that you can use at once in your wartime plans—with classes large or small, in cramped or unlimited space.

2. Part II contains 215 tested, effective skills games, with charts, covering 6 chief sports that adolescents like, and 2 sports that they will carry through adulthood.

3. The 7 chapters of Part I are devoted to the organization, administration, and teaching of the modern athletic skills program, according to accepted educational principles.

4. Here is the first book ever published on this most important problem in high-school physical education today.

Make wartime toughening exciting fun!

The 215 skills games in this book are big-muscle activities, that get pupils in shape for their wartime duties. They make exciting fun of a wartime requirement. And the 7 chapters of Part I show how all problems connected with a skills-game program can be solved. Throughout the book, there are suggestions on obtaining and improvising equipment on a small budget. Order a copy today for 10-day free examination!

10-day free examination—Net prof. price $2.20

INOR PUBLISHING CO. **207 Fourth Ave.
New York**

Index photographed at the beginning for the convenience of the microfilm user.

THE MATHEMATICS TEACHER

Volume XXXV DECEMBER · 1942 Number 8

CONTENTS

JUNIOR HIGH SCHOOL NUMBER

OFFICIAL JOURNAL PUBLISHED BY THE

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

MENASHA, WISCONSIN · NEW YORK, N.Y.

Entered as second-class matter at the post office at Menasha, Wisconsin. [illegible] February 28, 1925, [illegible] submitted March 1, 1930.

THE MATHEMATICS TEACHER

Official Journal of the National Council of Teachers of Mathematics

Devoted to the interests of mathematics in Elementary and Secondary Schools

[illegible]—WILLIAM DAVID REEVE, Teachers College, Columbia University.
[illegible]—VERA SANFORD, State Normal School, Oneonta, N.Y.
[illegible] W. S. SCHLAUCH, School of Commerce, New York University.

OFFICERS OF THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

President—R. R. SMITH, Springfield, Mass.
First Vice-President—R. L. MORTON, Ohio University, Athens, Ohio.
Second Vice-President—DOROTHY WHEELER, Hartford, Conn.
[illegible]—EDWIN W. SCHREIBER, Western Illinois State Teachers College, Macomb, Ill.
[illegible] *Representative*—KENNETH BROWN, 525 W. 120th St., New York City.

ADDITIONAL MEMBERS ON THE BOARD OF DIRECTORS

	[illegible] M. AUSTIN, Oak Park High School, Oak Park, Ill.	1943
One Year	[illegible], McMichael Intermediate School, Detroit, Mich.	1943
	[illegible] W. CHARLES [illegible], Denver, Colo.	1943
	KATE BELL, [illegible], Spokane, Wash.	1944
Two Years	J. O. HASSLER, University of Oklahoma, Norman, Okla.	1944
	L. H. WHITCRAFT, Ball Teachers College, Muncie, Ind.	1944
	A. B. CONNOR, Lincoln, Neb.	1945
Three Years	[illegible] H. HOLROYD, Manhattan, Kan.	1945
	A. BROWN MILLER, Cleveland, Ohio	1945

The National Council has for its object the advancement of mathematics teaching in elementary and secondary schools. All persons interested in mathematics and mathematics teaching are eligible to membership. The membership fee is [illegible] per year and entitles each member to receive the official journal of the National Council—THE MATHEMATICS TEACHER—which appears monthly except June, July, August and September.

Correspondence relating to editorial matters, subscriptions, advertisements, and other business matters should be addressed to the office of

THE MATHEMATICS TEACHER
525 West 120th Street, New York City (Editorial Office)

[illegible] THE MATHEMATICS TEACHER automatically makes a subscriber a member of the National Council.

SUBSCRIPTION PRICE [illegible] PER YEAR (eight numbers)

Foreign postage, 50 cents per year; Canadian postage, 25 cents per year. Single copies [illegible] cents. Remittances should be made by Post Office Money Order, Express Order, Bank Draft, or personal check and made payable to THE MATHEMATICS TEACHER.

PRICE LIST OF REPRINTS

[illegible]	1 to 4	5 to 8	9 [illegible]	13 to 16	17 to [illegible]	[illegible] to 24	[illegible]	[illegible]
[illegible]	$2.50	$4.00	[illegible]	$4.60	$ 8.55	$ 9.85	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	10.85	12.20	[illegible]	[illegible]
[illegible] per C	[illegible]	1.60	[illegible]	3.15	3.60	3.75	[illegible]	[illegible]

[illegible] THE MATHEMATICS TEACHER when answering advertisements.

16th Yearbook

of the

National Council of Teachers of Mathematics

ARITHMETIC IN GENERAL EDUCATION

The Final Report of the National Council Committee on Arithmetic

Recently voted one of the sixty best educational books of 1941

A redirection of arithmetic instruction toward meaningful comprehension on the part of the pupil, from his early years through the high school. Enrichment in the theoretical and practical phases of arithmetic is supplemented by substantial bibliographies of references and research studies.

CONTENTS

I. Introduction. By R. L. Morton.
II. The Function of Subject Matter in Relation to Personality. By G. T. Buswell.
III. Curriculum Problems: Grade Placement. By [illegible].
IV. Arithmetic in the Early Grades from the Point of View of Interrelationships in the Number System. By C. L. Thiele.
V. A Theory of Instruction for the Middle Grades. By Harry G. Wheat.
VI. Arithmetic in the Junior High School. By [illegible].
VII. The Social Phase of Arithmetic Instruction. By Leo J. Brueckner.
VIII. Utilizing Supplementary Materials and Devices in the Enrichment of the Arithmetic Course. By Irene Sauble.
IX. What Becomes of Drill? By B. R. Buckingham.
X. The Evaluation of Learning in Arithmetic. By William A. Brownell.
XI. Recent Trends in Learning Theory: Their Application to the Psychology of Arithmetic. By T. R. McConnell.
XII. Questions for the Teacher of Arithmetic. By F. Lynwood Wren.
XIII. The Interpretation of Research. By William A. Brownell and Foster E. Grossnickle.
XIV. One Hundred Selected Research Studies. By [illegible].
XV. One Hundred Selected References. By [illegible].

Price, [illegible] postpaid.

BUREAU OF PUBLICATIONS

Teachers College · Columbia University · New York

Please mention the Mathematics Teacher when answering advertisements.

17th Yearbook

of the

[illegible]tional Council of Teachers of Mathematics

A SOURCE BOOK
[illegible]F MATHEMATICAL APPLICATIONS

[illegible] by a Committee of the National Council of Teachers of Mathematics

[illegible] volume has been prepared to meet the need for a broader knowledge of direct [illegible]ication of mathematical principles. It provides a valuable reference book for teachers [illegible] mathematics usually offered in grades seven through twelve.

[illegible] four sections—Arithmetic, Algebra, Geometry, and Trigonometry—are treated [illegible]rding to mathematical topics alphabetically arranged. The numerous applications [illegible] topic are conveniently listed according to number. Hundreds of illustrations [illegible] material.

CONTENTS

[illegible]IC
[illegible] Average. Compound numbers. Decimals. Decimal point. Denominate numbers. Equa[illegible] Exponents. Formulas. Fractions. Fundamental operations. Graphs. [illegible] Interest. Measurement. Mensuration. Metric units. Numbers. Per cent. Proportion. [illegible] Scale drawing. Squaring a number.

[illegible]EBRA
[illegible]mulas. Fractional equations. Graphs. Linear equations. Progressions. Quadratic and other [illegible]tions. Variation and Proportion.

[illegible]TRY
[illegible], bisector. Angle, polyhedral. Angles. Areas. Circle. Cones. Conic sections. [illegible] drawings. Geometric forms. Geometry construction. Golden section. [illegible] circles. [illegible]allel lines. Parallelograms. Proportion. Pythagorean theorem. Rectangular solids. Regu[illegible]ons. Similar triangles. Spheres. Spherical angles. Spherical triangles. [illegible] Tangents. Triangles. Trisection of an angle. Volumes.

[illegible]OMETRY
[illegible] Cosine function of twice an angle. Cosines, laws of. Cosine ratio. Cotangent ratio. Radians. [illegible]s. Sines, law of. Sines and cosines, laws of. Sine ratio. Sine, versed. Tangent ratio. [illegible]etric functions. Trigonometric manipulations. Trigonometric tables. Trigonometry and [illegible]

$2.00 postpaid

BUREAU OF PUBLICATIONS

[illegible] College *Columbia University* *New York*

[illegible] mention THE MATHEMATICS TEACHER when answering advertisements